Understanding the Identities, Emotions, Attitudes and Motivations of Developmental Mathematics Students in the Context of Their Prior Learning and Life Experiences

by

Carolyn Pietrucha Masserang

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Educational Studies)
in the University of Michigan
2021

Doctoral Committee:

Professor Deborah Loewenberg Ball, Co-Chair
Professor Hyman Bass, Co-Chair
Professor Robert Megginson
Professor Vilma Mesa

Carolyn Pietrucha Masserang

cpmasser@umich.edu

ORCID iD: 0000-0003-3346-3629

© Carolyn Pietrucha Masserang 2021

Dedication

This dissertation is dedicated to

my new granddaughter

and all of my future grandchildren.

May you experience the joy of math.

Acknowledgements

This dissertation would not have been possible without the love and support of my husband, Keith. You have been so patient throughout this long process. Yes, I did say patient! You kept our home in order, provided technical support, and gave me much needed life balance while I pursued my passion. I am so grateful for you.

I would like to thank my co-chairs, Deborah Loewenberg Ball and Hyman Bass for their guidance, feedback, encouragement, and especially their kindness, as I made my way through the arduous dissertation process. Hy, thank you for supporting me throughout my entire doctoral process. You have always asked great questions that made me think more deeply. Your humble approach to teaching and mentoring has been inspiring. Deborah, before I knew what portraiture methodology was, you knew that if fit the goals of my study. I am grateful for your thoughtful guidance. I would not have this particular dissertation without your generously shared expertise and perspective.

I would also like to thank my committee members, Vilma Mesa and Robert Megginson. Vilma, as my professor you taught me what it really means to read critically, for which I will always be grateful. Thank you also for your careful reading and thoughtful feedback of my dissertation drafts. Bob, thank you for your encouragement and enthusiasm and for sharing your perspectives.

Thank you, Rebecca, for your feedback, exceptional editing and for moving me along. I would still be working on this without your help. I am thankful for my math education cohort

and extended cohort; I cannot imagine what this dissertation process would have been like without your support. I am especially grateful to Amber, Charles, Linda, and Nic.

I am also grateful to my School of Education professors for all that they have taught me about teaching, learning, and equity in education. I want to thank Michael Weiss for introducing me to mathematics education as a field of research. I would not have known of this program, let alone be part of it without your inspiration.

I would like to thank my participants who so generously shared many personal stories to help improve the system for the benefit of others. I also thank the Research Review Board for allowing me to collect data at their college. The promise of anonymity that I made to my participants included not using the name of the institution at which they were recruited. As such, I cannot use proper names but would like to thank the head of the mathematics department and the instructors who supported me. I would also like to thank the disability services coordinator who, in addition to heading-up a well-run office, was generous with her time. Thank you for helping me understand the needs of my students while I was an instructor at your school.

I would also like to express my appreciation and acknowledge the Rackham Graduate School for supporting me with a Rackham One-Term Dissertation Fellowship as I completed this dissertation.

And, finally, I thank my family—my children, by birth and by marriage, as well as my siblings, and again, my husband. Thank you for listening to me talk too much about my work, especially Melissa and Joan as my sounding boards and Daniela as my writing partner. My parents passed away during my time in this program. I am grateful for their sacrifices and for giving me a lifetime of privileges that were mostly invisible to me until I started this program.

Table of Contents

List of Tables

List of Figures

List of Appendices

Abstract

Low achievement in mathematics begins long before students arrive at college. National data shows that between 60% and 75% of high school seniors in the U.S. are underprepared for college-level mathematics and may end up enrolling in developmental mathematics. Developmental mathematics refers to mathematics courses taken in college that are considered to be remedial, or below the level of college mathematics. These courses have the potential to provide educational access to those who have been underserved in their K-12 education. To help improve students' outcomes in these classes, interventions have been proposed and implemented, but with limited success. I argue that the problem with these interventions is that the field knows very little about who developmental mathematics students are as math learners. This is because research on these students is limited and their voices are largely missing.

Students bring more than knowledge of mathematics with them to their college classes. They bring memories of past experiences that continue to influence their learning. They also bring noncognitive aspects of learning such as identity, emotions, attitude and motivation, collectively referred to as mathematics-related affect. In this dissertation study, I use portraiture methodology to explore the memories of 10 developmental mathematics students to understand how their prior learning and life experiences shaped their relationships with mathematics. Data consisted of a background questionnaire and a one-on-one interview with each participant. I developed a framework to analyze my data that positions the narrative-identity framework of Sfard and Prusak (2005) at the center of mathematics-related affective characteristics as well as students' life and learning experiences related to mathematics learning.

I present my findings in the form of portraits of these four participants along with summaries of the other six. Each participant talked about each aspect of my framework. However, the data for each highlighted the importance of different aspects: the role of emotions in motivation for learning mathematics; the ways that one's beliefs about the importance of math interact with identity; the relationship between motivation and attitude; and the way that environment impacts learning and influences all aspects of affect.

I organize my findings around major themes that I noticed in my data. Perhaps the most important theme was participants' development of strategies for self-protection. They described a variety of strategies they developed to protect themselves from harm usually in response to negative emotions such as anxiety. These strategies were sometimes misinterpreted by authority figures. In addition, I found ways that my participants were resilient and persistent, even as the literature on developmental mathematics often positions them as failures. My analysis also illustrates that a reciprocal relationship exists between each aspect of mathematics-related affect, as well as the importance of considering the environment in which these affective characteristics develop. Findings suggest that there are many ways that a students' actions might be misunderstood by faculty and that it's vital to consider students' point of view when providing instruction and other resources. I discuss the implications these findings and my revised conceptual framework have for future research and teaching in developmental mathematics.

Chapter 1
Introduction

Writing this dissertation has been a humbling experience. When I taught developmental mathematics courses (also referred to as remedial) to college students, I often made it a point to get to know my students as people and as math learners within the boundaries of being their instructor. For this dissertation, I devised a plan to move from anecdote to research so that I could write about the ways that students' past learning and life experiences influenced who they are as math learners when they arrive at college. As a researcher, I could ask questions that would have been inappropriate for me to ask a current student. I expected that as I collected and analyzed data from developmental mathematics students as research participants, I would learn more deeply about things I had already heard about from or had noticed about my past students. Although this did happen to some extent, I also learned about things I had not even known to look for.

As an example, if someone had asked me if people develop strategies for self-protection, I would have answered, "Well yes, of course." If someone had asked me if students are reluctant to ask questions in class, go to the tutoring center, or to office hours because they felt embarrassed and didn't want someone to think they are not intelligent, again, I would have felt that this was obvious. I don't imagine anyone likes to feel judged. I know I don't like that feeling. But I have not had learning and life experiences that lead me to expect that I *will* be judged and humiliated when I enter the mathematics classroom or the tutoring center. I did not

expect to find that, for *some* students, self-protective strategies are one of the most important things that they bring with them to their developmental mathematics courses.

I often wondered, given that pass rates in developmental mathematics are typically significantly lower than in higher level mathematics courses, why the tutoring center is not regularly filled with developmental mathematics students. On a more extreme level, why would someone rather *fail* a class than go to the tutoring center or to office hours for help? The key word here is *rather*. I now realize that my question came from a place of privilege. I did not anticipate that some of my participants would have a history of repeated humiliation associated with mathematics. The tutoring center and the instructors' office area are public spaces; spaces that can be resources for some but may be places of perceived harm to another. My question has evolved to under what circumstances *would* someone, who has a history of being humiliated when asking for help, take the chance of it happening again? Where would such resilience come from?

Teaching developmental mathematics courses opened my eyes to some of the great inequities in our education system. I met students from walks of life that are very different from my own. I have heard many stories of hardship and limited opportunities for learning. I previously had no idea that I have lived a very privileged life.

My interest in understanding how past context impacts developmental mathematics students began during one of my first semesters teaching developmental mathematics. I had a student in Elementary Algebra whose test and quiz grades were in the single digits out of one hundred points at the start of the semester. As a new instructor, I did not initially think that she could be "successful." But what is success? She made a friend in the class; they worked together at appropriate times during class, did homework together, came to office hours together, came up

before and after class with questions, and went to tutoring together. Her grades steadily improved and she scored just over 50% on the final exam! Of course, 50% is not high enough to pass the course, but by the end of the semester, she had learned enough to be in a position to retake the class and receive a passing grade the following semester. Years later, while I reviewed the research literature on developmental mathematics, I found that much of the literature positions as failure what was, for her, an enormous triumph.

Developmental Mathematics

In this section I provide a brief description of what is meant by "developmental mathematics," the state of research on it and how it is commonly viewed in the literature. I describe each of these in greater detail in Chapter 2.

Definition

Developmental mathematics refers to mathematics courses taken in college that are considered to be below the level of college mathematics. But what is considered to be a college level course varies by institution. College Algebra and Pre-Calculus are often considered to be the lowest level college mathematics courses, although some institutions consider Intermediate Algebra to be college level (Blair et al., 2006). Courses below this level are often referred to as "developmental." These courses cover mathematics that students are commonly expected to have learned by the end of high school. However, there is no universally accepted sequence. The typical course progression in 2010 started with Arithmetic, followed by Pre-Algebra, and Elementary Algebra and ended with Intermediate Algebra. Since then, Arithmetic has been eliminated at many institutions (Blair et al, 2018). Some institutions refer to Intermediate Algebra as "transitional." In this study, I refer to Pre-Algebra and Elementary Algebra as

developmental and Intermediate Algebra as transitional. I present this in greater detail in Chapter 2.

What Do We Know?

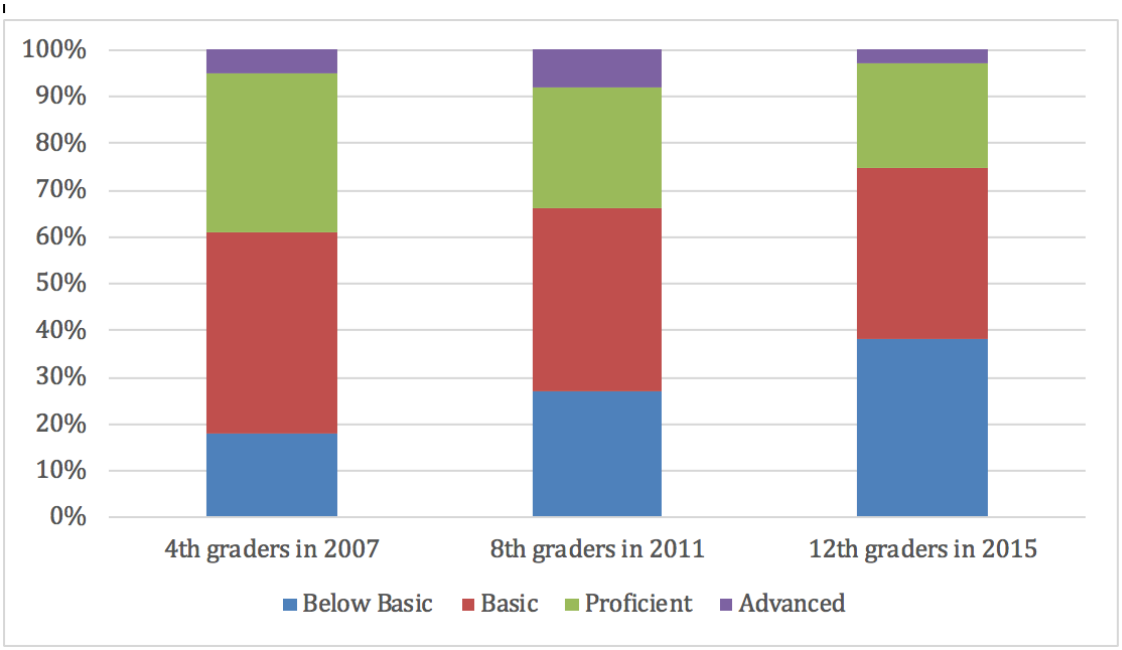
Research on developmental mathematics courses and students is limited and fragmented. Much of the research narrowly defines success as progression through a sequence of courses ending with a passing grade in a college level course. Authors commonly present data from a negative point of view. According to Blair et al. (2018), since 2005 well over one million students have been enrolled in developmental and transitional mathematics courses each year. Much of the research cites statistics such as the numbers of students enrolled, but research tells us very little about who these students are as math learners.

Inequity in K-12 Preparation for College-Level Work

National Center for Education Statistics (NCES) provides data on student achievement in mathematics in terms of the percentages of students considered to be at an advanced, proficient, or basic level, or below the basic. This data is collected on a nationally representative sample of students in 4th, 8th and 12th grades. Students who were in 12th grade in 2015 were in 8th grade in 2011 and in 4th grade in 2007. In 2007, the percentage of those 4th graders who were advanced, proficient, basic and below basic was 5%, 34%, 43% and 18%, respectively. In 2011, the percentage of 8th graders who were advanced, proficient, basic and below basic was 8%, 26%, 39% and 27%, respectively. And 12th grade students in 2015 were 3%, 22%, 37% and 38%, respectively. (McFarland, et al., 2018, p, 109). So, in 2015, only 25% of high school seniors scored proficient or above and more than one-third of students scored below the basic level. This is illustrated in Figure 1.1.

Figure 1.1

Percentage of 4th, 8th, and 12th Grade Students, From a Nationally Representative Sample, Who Scored in the Advanced, Proficient, Basic, and Below Basic Range



Note. Figure 1.1 uses data from NCES (McFarland, et al., 2018, p. 109).

In 2015, 69% of recent high school graduates enrolled in college; 25% enrolled in 2-year and 44% enrolled in 4-year institutions (McFarland, et al., 2018, p. xxv-xxvi). This does not include older returning students who may have been out of school for many years and need to take refresher courses. Based on this data, one might expect that the average high school graduate would not be prepared for college level mathematics.

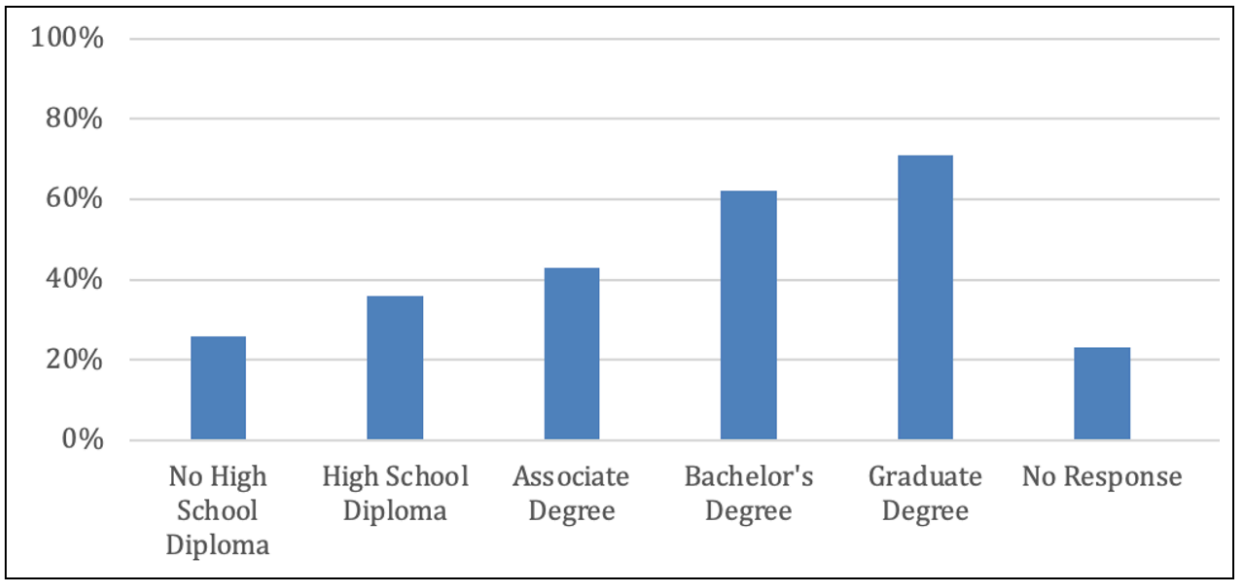
Other measures of college-readiness include ACT and SAT sores. ACT, Inc. and the College Board (makers of the SAT) have established benchmark scores to indicate that a student is ready for college-level mathematics. Both test makers predict that students who meet their benchmark have a 50% chance of achieving a B or higher and a 75% chance of achieving a C or higher in a college-level course such as College Algebra (ACT, 2017; College Board, 2018). In 2017, 41% of students who took the ACT met the mathematics benchmark (ACT, 2017). Also, in

2017, 46% of students who took the SAT met the mathematics benchmark (College Board, 2018). However, the College Board identified what they consider to be a nationally representative sample of all high school students out of those students who actually took the test that year and found that, had all students taken the test, only 39% of this sample were college ready in mathematics (College Board, 2018).

Discrepancies by parents' highest education level and by race highlight inequitable K-12 educational opportunities students receive prior to college enrollment. Figure 1.2 shows that 26% of test takers whose parents did not graduate from high school met the benchmark, while 71% of test takers who had at least one parent with a graduate degree were deemed college ready in mathematics (SAT, 2017).

Figure 1.2

Percentage of Test Takers Meeting Mathematics Benchmark Scores on the SAT in Mathematics by Highest Level of Parental Education at the National Level.

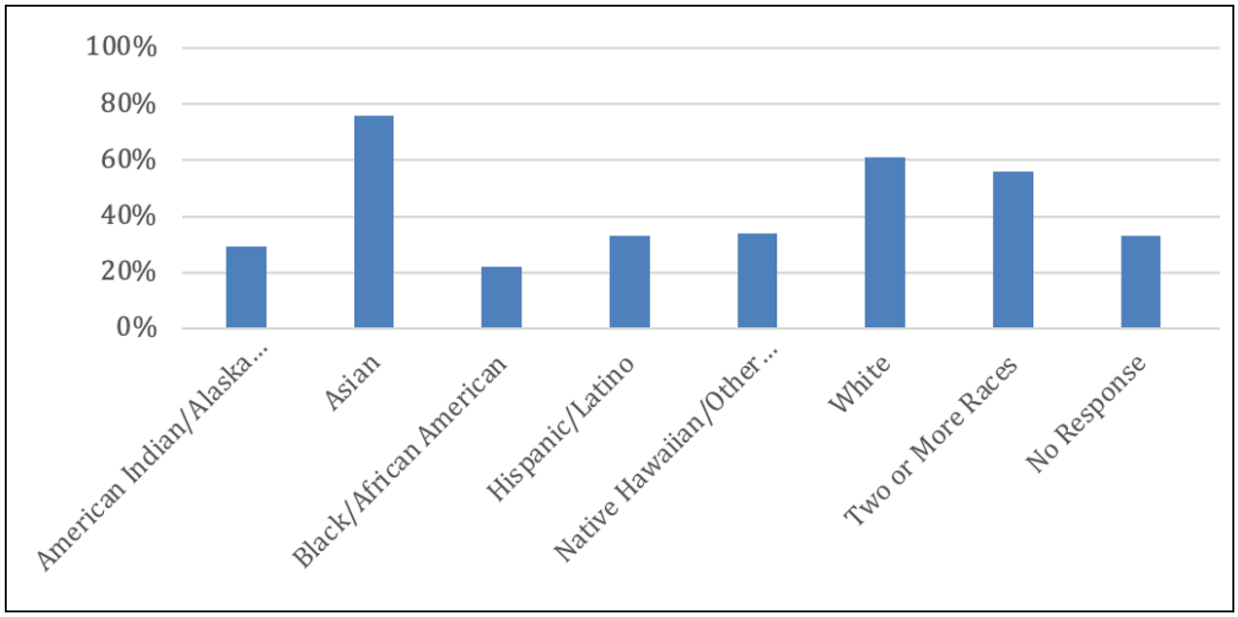


Note. Figure 1.2 uses data from SAT Suite of Assessments Annual Report (SAT, 2017).

SAT data also shows discrepancies by race. More than 50% of students identifying as Asian, White and Two-or-More Races scored at or above the benchmark scores while fewer than

50% of students who identified as American Indian/Alaska Native, Black/African American, Hispanic/Latino or Native Hawaiian/Other pacific Islander scored at or above the benchmark, as shown in Figure 1.3.

Figure 1.3
Percentage of Test Takers Meeting Mathematics Benchmarks Scores on the SAT in 2017 by Race/Ethnicity at the National Level.



Note: Figure 1.3 uses data from SAT Suite of Assessment Annual Report (SAT, 2017).

I will show in greater detail in Chapter 2 that college students have been described as underprepared for college-level work for centuries. I will also show, the state of research on developmental mathematics.

Problem Statement

As the testing data shows, inequities in educational opportunity and achievement begin during students' K-12 years and carry-over to college enrollment. Colleges do not create the problem of low student achievement, but they are charged with educating students who were previously underserved. At the same time, research on how to best educate developmental

mathematics students is sparse. Rather than improve teaching and learning to provide greater opportunity, financial aid is being reduced and some states are eliminating developmental mathematics; the trend is to deny students who did not have a fair first chance during K-12 any opportunity to learn mathematics during college. This leads me to think of the invisibility of privilege. It may be difficult for those who, like me, have had the benefit of good teaching and an appropriate level of rigor in their mathematics education to understand that many developmental mathematics students did not have the chance to learn mathematics in a meaningful way prior to college enrollment.

After I completed the coursework portion of my doctoral program, I returned to teaching developmental mathematics part-time while I worked on my dissertation. But what I was reading about developmental mathematics students in the literature did not match the reality of what I was seeing in the classroom. I had students who persevered and took the same class two or three times before passing. I also had students who, after having a difficult time in either Elementary Algebra or Intermediate Algebra, decided to drop the class and take a lower level class to fill gaps in their mathematical learning, before continuing on to complete their required mathematics courses. I did not find this type of perseverance acknowledged in the literature. I also learned from my students that it was often the case that varying levels of developmental mathematics courses fulfilled the mathematics requirement for the degree or certificate they were pursuing, at the same time that I was reading literature that views not taking College Algebra or Pre-Calculus as a failure.

Many students shared stories of various past experiences that illustrated how developmental mathematics was not their second chance at learning mathematics in a meaningful way, it was their *first* chance. Most students described being taught mathematics as rote

procedures with no understanding throughout K-12. Some students missed years of schooling because of abuse or numerous foster care changes. One student described losing his hearing so slowly that his hearing loss was not recognized until he was completely deaf. He was labeled as low-intellect even after his diagnosis and was placed in mathematics classes below his ability level throughout high school. Many students talked of being underestimated in terms of mathematical ability, being placed in low-level unchallenging classes, and receiving social promotions rather than support and an appropriately challenging curriculum. Yet, this is not a significant part of the national conversation.

Although I had the privilege of seeing some students make great strides in learning, I did not see this in all students. I felt underprepared for teaching this student population. I searched the literature for a better understanding of who developmental mathematics students are as math learners and for literature about how to teach K-12 math to adult students. But I found limited research on how to better serve this population; instead I found that developmental mathematics courses are being eliminated.

Shalaby (2017) talks about the ways that some young children are treated in school as though they have *throwaway lives*. By choosing to eliminate and reduce funding for developmental mathematics courses are we not treating developmental mathematics students as having throwaway lives as well? As a matter of equity, we must begin to understand who they are as math learners as a first step to provide improved learning experiences.

Placement tests tell us, at most, how much mathematical knowledge and understanding a student possesses as they start college. Mathematics-related affective characteristics such as identity, emotions, beliefs and motivations, along with past learning and life experiences also impact learning and must be accounted for in the teaching and design of coursework as well as

student supports. I argue that we cannot know these students as mathematics learners without an understanding of how their past learning and life experiences have shaped them as math learners and an understanding of the range of emotions and beliefs that they bring with them to college.

Purpose

The purpose of my study is to learn directly from students about the ways that their learning was impacted by their learning and life experiences prior to enrolling in developmental mathematics courses and to begin to understand the mathematics-related affective characteristics of identity, attitude, emotions and motivations that they bring with them. I wanted to understand how their past experiences in combination with these characteristics impacted their actions such as doing homework, engaging in class and asking for help when needed. Understanding who they are also includes knowing about the ways that the opportunities and barriers they experienced shaped them as math learners. I also wanted to know how they define success in mathematics as well as which topics were the most difficult for them. I wanted to learn about these things directly from students themselves and to present their perspectives which are rarely included in the developmental mathematics literature.

Research Question

The research question that guides this study is:

What do developmental mathematics students' accounts of their past learning and life experiences reveal about them as math learners that their grades do not?

Overview of Research Design

After I described the goals of my study to my co-chairs, one of them handed me the book *Troublemakers* by Shalaby (2017) which uses the methodology of portraiture to present the

students' view of schooling. I was drawn to a particular boy in her study who learned at home that people should help each other. But his teacher wanted independent learners and he was punished in school for trying to help other students. I do a lot of small group work in my classes. It typically goes well, but sometimes, some students are resistant to working this way. I would not assume that any particular developmental mathematics student experienced this type of schooling, but it offered me a possible explanation. It allowed me to be more understanding of students who may be uncomfortable working this way.

Like Shalaby (2017) I wanted to express the students' points of view and offer counterexamples to the prevailing views of the literature which often use the word "failure" in the context of developmental mathematics. I wanted to look deeply at each student as an individual before considering my participants collectively. The methodology of portraiture, which I describe in much more detail in Chapter 3, allowed me to do that.

Portraiture requires strong relationships with research participants. I first invited some former developmental mathematics students of mine which allowed me to build on prior relationships in a research setting, so that their stories may be analyzed and told. Eight of my former students participated in the study. I also recruited students from two other instructors' classes, for a total of 10 participants.

To design my study and guide data collection, I used portraiture together with a conceptual framework that builds on the narrative-identity framework developed by Sfard and Prusak (2005). According to this framework, identities are human-made, not inborn, and develop over time. Identities are words that are taken seriously and that shape our actions. So, stories of what a person does, are also an indication of their identity. But as I began to analyze data, I found that identity alone did not provide a sufficient framework for understanding the way that

my participants experienced their past learning. So, I redeveloped my conceptual framework. This new framework maintained the narrative-identity framework but positioned it at the center of other aspects of mathematics-related affect in the context of learning and life experiences. I used this new framework together with portraiture methodology to analyze and present data.

Data Collection

Each participant was asked to fill out a written background questionnaire prior to an in-person interview. Interviews were semi-structured and included follow-up questions associated with background questionnaire responses as needed, as well as a profile sketch. On the profile sketch (Gholson & Robinson, 2019), students were asked to write messages they heard from others, both in-school and out-of-school as well as messages they told themselves. Typically, three sketches were filled out, one each for elementary school, middle school, and high school. Participants were encouraged to add anything they felt was important to know that I did not ask about.

Chapter 2
Review of Relevant Literature and Conceptual Framework

Developmental or remedial education has been a regular part of U.S. education since colonial times; coursework in mathematics for underprepared college students dates back to the mid-19th century (Breneman & Harlow, 1998). However, rigorous research on teaching and learning in developmental mathematics is in the beginning stages (Mesa, 2017). We know very little about who these students are as math learners, a precondition for effectively developing programs and services to improve instruction and learning.

I begin this chapter with an overview of the historical development of remedial education in U.S. colleges and universities, as well as postsecondary enrollment trajectories. Then I turn to the recent literature, with a specific focus in mathematics. I describe the state of current research, the changing and sometimes conflicting views of the role of developmental mathematics and the ways that various authors define "success." Next, I consider calls for future research, in particular, the need for qualitative studies of students' experiences and perspectives. I then discuss literature on emotions, beliefs and motivation of math learners, collectively referred to as mathematics-related affect, before focusing on the narrative identity framework developed by Sfard and Prusak (2005). Finally, I describe my conceptual framework, that centers on narrative identity but includes other aspects of mathematics-related affect as well as the impact of prior experiences and chances to learn.

Remedial/Developmental Education

Historically, the term "remedial," has been used to describe a range of courses and tutoring aimed at helping students prepare for college-level work. Although many authors treat remediation as a recent phenomenon associated with community colleges, remediation actually began in the Ivy League institutions and has been a regular part of U.S. higher education since before we were a nation (Boylan & White, 1987; Breneman & Haarlow, 1998; Merisotis & Phipps, 2000). However, in the past several decades, as programs and services have been developed to incorporate additional supports for students (e.g., student success courses, tutoring centers, assessment, placement, advising and counseling), the term "developmental" has been used more frequently (Breneman & Haarlow, 1998; Kozeracki, 2002; Payne & Lyman, 1996). Across the literature, authors have used the terms "developmental," "remedial," "non-credit," and "precollege" somewhat interchangeably, but the most recent literature uses the term developmental. Much of the literature on developmental mathematics is intertwined with developmental education, which in recent literature includes reading and writing. Here, I offer a brief overview of the history of remediation in higher education to provide the reader with a context for situating the current state of developmental mathematics.

Historical Origins of Developmental Education

Remediation initially took place in the form of tutoring affluent White male students in Greek and Latin at Harvard College in the 1630s (Boylan & White, 1987; Breneman & Harlow, 1998; Merisotis & Phipps, 2000). At this time, it was seen as a natural part of higher education. Eventually, as demand for higher education grew, the need for remediation became so great that institutions began to create remedial courses, including in mathematics, in the mid-19th century (Breneman & Haarlow, 1998; Meserve & Schumaker, 1957; O'Quinn, 1931; Reeve, 1946;

Williams, 1954). By 1889, an estimated 80% of U. S postsecondary institutions had programs for remediation (Canfield, 1889 as cited in Boylan & Hunter, 1988) and maintained preparatory departments throughout the 19th and into the 20th century (Breneman & Haarlow, 1998). In her review of college reading courses, Wyatt (1992) found that more than half of students at Harvard, Yale, Princeton, and Columbia in 1907 did not meet their schools entrance requirements and added developmental courses.

Rise in Postsecondary Enrollment and Demand for Developmental Education

A rise in the demand for postsecondary education that occurred during the 20th century (NCES, 2020) was accompanied by a rise in the need for remediation which was unexpected by professors as well as by students who thought they were prepared for college. For example, Williams (1954) found that during World War II, students from all parts of the country, who were stationed at the University of South Carolina, were unprepared for college-level mathematics. Remedial non-credit courses in algebra and geometry were introduced with the intention of dropping them after the War. It was assumed that the "tenseness of the times brought on by the war" (p. 86) was the cause. However, after the War ended, the need for the remedial courses continued. Williams also found that the underprepared students thought that they were prepared when they arrived at college.

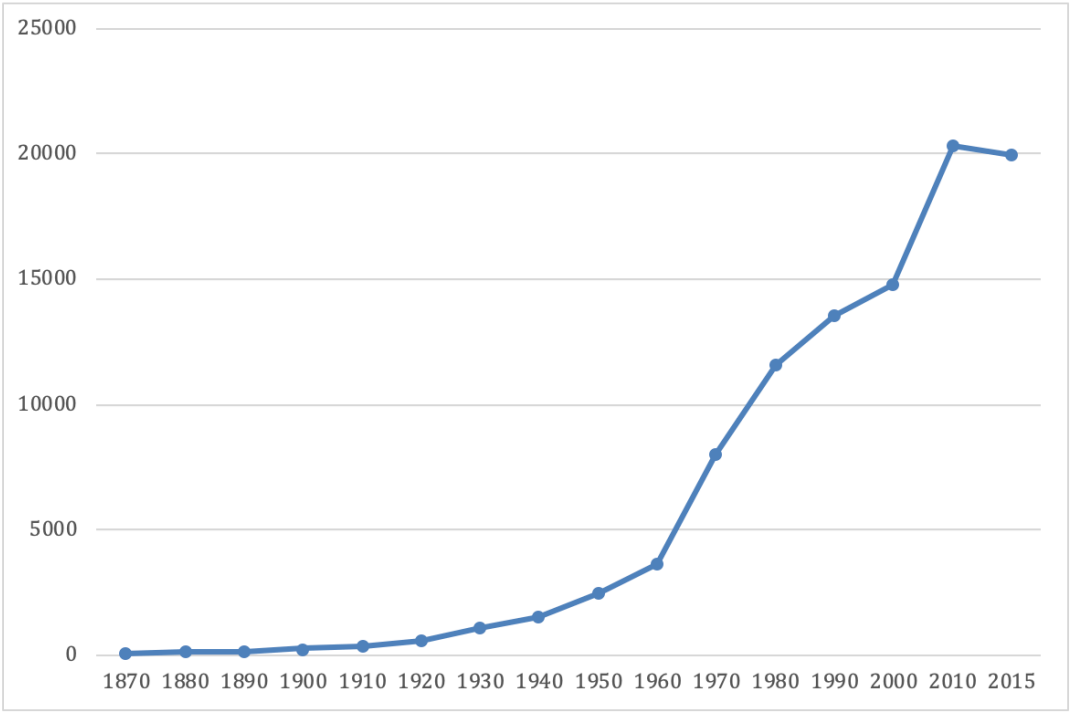
His concerns about the underpreparedness of his students led Williams to conduct a survey to determine if other colleges and universities were experiencing a similar situation and what they were doing about it (1954). His department sent out over 500 questionnaires to postsecondary institutions across the United States. Among the respondents, only 53% felt their college algebra students were good or fair, with the remainder classified as either poorly or totally underprepared. Further, 41% of respondents had started to require placement tests, 62%

provided remedial courses, and 61% of those who provided a remedial course allowed credit for it. The survey also showed that 34% of students at institutions represented in the survey were enrolled in a remedial algebra course. However, the percentage of students who were underprepared for college mathematics was likely greater because, according to the respondents, many students who placed into a remedial course did not take it; they either changed to a curriculum that did not require mathematics or dropped out of college. An overwhelming majority of respondents felt the best solution was for high schools to cover less material and to cover it more thoroughly. Williams concluded that the problem of high schools not preparing students for college level mathematics was not new and that the War brought a problem that already existed to the surface.

The 20th century saw a dramatic rise in the numbers of students attending college, according to the National Center for Education Statistics (NCES). Figure 2.1 shows the rise in the total number of students enrolled at degree-granting institutions from the late 19th into the early 21st centuries.

Figure 2.1

Total Enrollment in the Thousands at all Degree-Granting Institutions from 1870-2015



Note. Figure 2.1 uses data from the National Center for Education Statistics (NCES, 2020).

Federal financial support for developmental courses began with the Servicemen's Readjustment Act of 1944 (Wyatt, 1992). Veterans taking advantage of what is more commonly referred to as the G.I. Bill created a surge in the need for remedial education (Merisotis & Phipps, 2000). During the late 1960s and early 1970s, demand for developmental courses surged again as Baby Boomers entered postsecondary education.

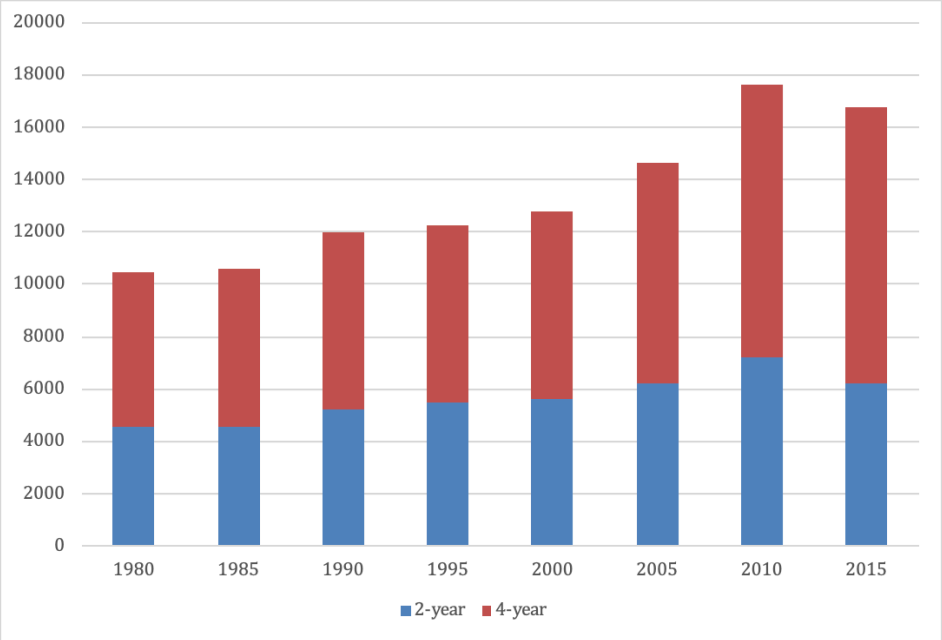
Payne & Lyman (1996) found that when underprepared Baby Boomers "came to college, significant amounts of money were allocated for delivery of basic skills programs. However, in the rush to take services to the most needy students, research and program evaluations were assigned a low priority" (p. 15). Wyatt (1992) found in her review that during this period of increased demand, courses in these programs often lacked rigor and were based on elementary

school courses that were poorly redesigned for adults. During this period, 90% of students in developmental courses at open admissions colleges either failed or withdrew.

The Role of Community College Expansion on Postsecondary Enrollment

The expansion of community colleges contributed significantly to the increase in overall college enrollment. During the late 1960s and throughout the 1970s, the number of open-admissions two-year colleges expanded which offered students an important educational alternative (AMATYC, 2020). The attraction of community colleges was not limited to their open-admissions policies; the fact that they were local institutions "did more to open higher education to more people than did its policy of accepting even those who had not done well in high school" (Cohen & Brawer, 1996, p. 16). In particular, during the 1950s and 1960s as community colleges opened in parts of the country that did not previously have a local publicly supported college option, the proportion of recent high school graduates who enrolled increased, sometimes up to 50 percent. By the mid 1970s, "34 percent of all students enrolled in U.S. higher education institutions were enrolled at community colleges (Cohen & Brawer, 1996, p. 32). After more than a century of continuous increases in enrollment, undergraduate college enrollment reached an all-time high of 17.6 million students during the fall of 2010. At that time, 7.2 million, or 41% of all college students were enrolled at a two-year college (NCES, 2020). See Figure 2.2, below, for total undergraduate enrollment at two-year and four-year institutions every five years from 1980 through 2015.

Figure 2.2
Total Undergraduate Enrollment (In Thousands) at Two-Year and Four-Year Degree Granting Institutions from 1980 - 2015



Note. Figure 2.2 uses data from the National Center for Educational Statistics (NCES) Table 303.70. (NCES, 2020).

Racial Minority Enrollment

In addition to the massive expansion in overall postsecondary attendance, by the end of the 21st century the demographic make-up of *who* attended had begun to shift. The ostensibly race-neutral G.I. Bill was, in fact, used to exclude Black veterans from higher education and widen "an already huge racial gap [in educational attainment] in postwar America" (Humes, 2006; Martin, 2013; Katznelson, 2005, p. 121). Thus, through the middle of the 20th century, the expansion in college enrollment and concomitant demand for remedial courses was driven by huge increases in the number of White students.

During the fall of 1976, only 16% of students who were enrolled at all postsecondary institutions were ethnic minority students; during the fall semesters of 1990, 2000 2010 and 2015, ethnic minority students accounted for 20%t, 29%, 37% and 42%, respectively. Between

2010 and 2015, as overall postsecondary enrollment declined by 14%, minority enrollment increased by 6% (NCES, 2019).

Shift in Location of Remedial Course Offerings

As the community college alternative increased opportunity for enrollment, it further contributed to the increase in developmental course offerings (Wyatt, 1992). These "junior" colleges (later called community colleges) began to take on the role of remediating students, although many four-year schools continued to provide remedial programs (Breneman & Haarlow, 1998). By the turn of the 21st century, 70% of students who were enrolled in a developmental mathematics course were enrolled at a two-year college (Blair et al., 2018). Prior to the turn of the century, some states began to consider policies that shifted responsibility of developmental courses to the community colleges (McMillan et al., 1997). So, policy as well as open-admissions contributed to the enrollment shift in developmental course-taking from four-year to two-year colleges.

Changing Purpose of Developmental Education

The original purpose of developmental education which began in the four-year colleges and universities was to prepare students for college-level work toward the completion of a bachelor's degree. But community college students do not always have the goal of a bachelor's degree. Using data from the Beginning Postsecondary Students Longitudinal Study of 1996-2001, Bailey et al., (2005), found that 40% of community college students stated that they enrolled because "they wanted job skills or personal enrichment," and did not necessarily have degree or certificate attainment as a goal (p. 8). The 2013 chair of the Developmental Mathematics Committee of the American Mathematical Association of Two-Year Colleges (AMATYC) described three goals of developmental mathematics in community colleges: (1) to

prepare students for academic and life success, (2) to prepare students for a variety of college math courses, and (3) to prepare students for other courses with quantitative needs (Rotman, 2013).

Turn of the Century Controversy and State Policy Considerations

Providing developmental courses became controversial at the institutional, state and national levels near the turn of the 21st century (Kozeracki, 2002). Adelman (1999) noted that a significant development in higher education during the 1990s was:

> the growing public use of institutional graduation rates as a measure of accountability, and the tendency in public policy and opinion to blame colleges for students' failure to complete degrees and/or for failure to complete degrees in a timely manner (p. v).

Prior to the turn of the century, states had already considered policies that included prohibiting state money from being used to pay for developmental coursework and requiring K-12 public schools to reimburse colleges for developmental coursework needed by their graduates. States had also already considered limiting developmental coursework to the freshman year, limiting the number of courses offered, and concentrating it in the community colleges (McMillan et al., 1997).

What is striking is that, although educators have been concerned about underprepared students for decades (e.g. Williams, 1954), and remediation has been part of U.S. postsecondary education for centuries, public policy debates about the cost and efficacy of remedial/developmental programs did not become central to the discourse until the student body became more racially diverse, accountability debates for degree completion began, and disagreements over the purpose of developmental education arose.

Developmental Mathematics

In this section, I specifically consider literature on developmental mathematics, beginning with current views as to what courses make-up developmental mathematics. Next, I show that research interest in developmental mathematics is evolving but that recent policy proposals impacting developmental mathematics course offerings predate the beginnings of peer-reviewed research publications. Then, I discuss trends in the current literature before I situate my study in that literature in consideration of calls to research by the American Mathematical Association of Two-Year Colleges.

Definitions(s) of Developmental Mathematics

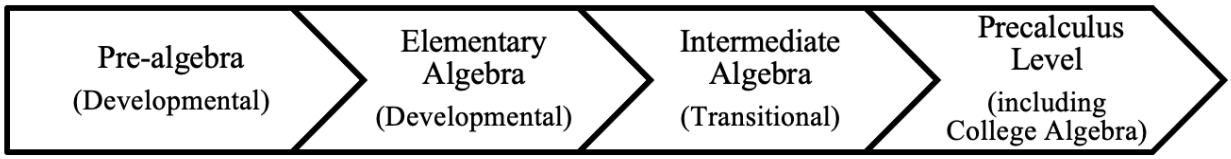
Currently, there is a lack of consensus as to specifically which courses are considered developmental. When developmental mathematics courses were first offered in the mid-19th century, remedial mathematics was arithmetic (Breneman & Harlow, 1998). Today, the American Mathematical Association of Two-Year Colleges (AMATYC), which is the professional organization for community college mathematics instructors, defines developmental mathematics courses as: "courses below the level of the first mathematics course that earns full college credit at the institution. For most two-year colleges, this includes mathematics courses below the level of intermediate or college algebra" (Blair, 2006, p. 41). However, Miami Dade College, the largest community college in the U.S., classifies Intermediate Algebra as a "transitional" course (Miami Dade College, 2019). Further, community colleges in the state of Florida refer to Intermediate Algebra as a "gateway" course (Hu et al, 2016).

According to recent surveys by the Conference Board for the Mathematical Sciences (CBMS), the typical course progression from the lowest level course to College Algebra, historically began with Arithmetic & Basic Mathematics. However, Arithmetic & Basic

Mathematics is being phased out and there has been a 52% drop in the number of colleges offering this course between 2010 and 2015. Some schools have also offered a developmental course in Geometry in the past, but fewer than 1% of students have been enrolled in Geometry since 2000 (Blair, et al, 2018).

In this study, I use the term "developmental mathematics" to include Pre-algebra and Elementary Algebra, and the term "transitional" to refer to Intermediate Algebra. Figure 2.3, below, illustrates the progression of the remaining developmental and transitional mathematics courses leading up to courses at the Precalculus level, including College Algebra.

Figure 2.3
Basic Mathematics Course Progression from Developmental Level to Precalculus

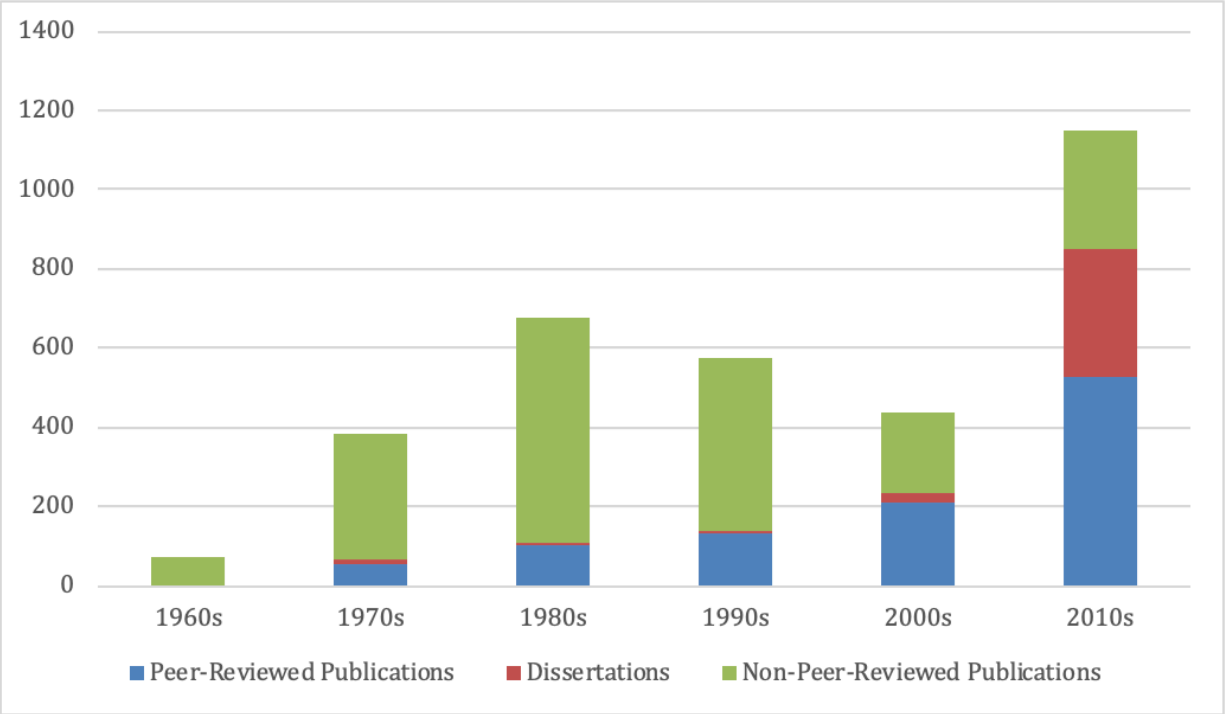


Research Interest in Developmental Mathematics

While developmental mathematics courses are not new, until relatively recently, most research on developmental mathematics was not published in peer-reviewed publications. During the last decade, there has been a significant increase in the number of peer-reviewed publications as shown in Figure 2.4. This increase in scholarly research interest corresponds to an enrollment spike in 2010 when 1,359,000 students were enrolled in a precollege level mathematics courses at two-year and four-year institutions combined, 1,150,000 of whom were enrolled at a two-year college (Blair et al., 2018). I also found that much of the research was focused on developmental mathematics at community colleges which makes sense because most students who take these courses do so at a community college.

Figure 2.4

Numbers of Publications on Developmental Mathematics Education Research in Peer Review Publications, Dissertations and All Other Non-Peer Reviewed Publications



Note. Figure 2.4 is based on an advanced search of ERIC (Educational Research Information Center) ProQuest using the key words (mathematics OR math) AND (developmental OR remedial OR precollege OR noncredit) AND (college OR university OR postsecondary OR undergraduate OR "higher education") in August 2020. Results for all publication types include scholarly journals, books, dissertations & theses, reports, conference papers & proceedings as well as encyclopedias & reference works (ERIC, 2020).

I also found that serious study of developmental mathematics is in its early stages and that the recent peer-reviewed literature has its roots in the existing non-peer-reviewed literature. My review of these bodies of work found that, rather than investigating mathematics students' experiences in developmental courses, authors commonly focused instead on narrow definitions of success tied to certificate and degree completion rates, and negative framings of the efficacy of these courses. I also found fragmented research on interventions and reforms, often with

conflicting results as I describe below, as well as calls for increased coordination between high schools and colleges.

In her synthesis of research conducted on mathematics education at U.S. public two-year colleges, Mesa (2017) found that this research "is in its infancy" (p. 962). This author also found that much of the research on community college mathematics has been done by the higher education community with a focus on success defined as students' completing degrees and passing courses, rather than success as learning (Mesa, 2017).

Trends in the Current Literature

Focus on Completion Rates

Much of the research literature comprises quantitative research that uses students' transcript data to narrowly define success as persistence through a sequence of postsecondary mathematics courses in a specified number of years (e.g., Bahr, 2008; Bailey, Jeong & Cho, 2010; Carnegie Foundation, 2016; Cullinane & Treisman, 2010; Visher, Cerna, Diamond, & Rutschow, 2017; Williams & Siwatu, 2017), or in terms of grades in individual courses (e.g., Fong, Melguizo & Prather, 2015). This narrow definition of success does not account for students' learning experiences or the role of prior obstacles to learning that students in developmental mathematics may have experienced. Nor does it acknowledge students' individual goals, which might not include taking the final course in a sequence. It does not recognize students' learning and accomplishments that may not be reflected in the final course grades. For instance, course grades below "C" are typically interpreted as failure, even though for particular students, a course grade approaching a "C" may represent great strides toward transitioning to college-level work.

Negative Views

I found many instances of developmental mathematics as well as developmental education more generally presented in a negative light. Non-peer-reviewed publications include numerous briefs and articles from research centers including the Community College Research Center (CRCC), Carnegie Foundation for the Advancement of teaching (CFAT), and MDRC. These organizations tend to publish either internally or in journals outside of education journals. Publications by these organizations tend to use the word "failure" to describe developmental courses, In particular, reports, articles and website information authored by CFAT associates have referred to developmental mathematics in a demoralizing light as "where dreams go to die," and "where aspirations go to die" (Carnegie Foundation, 2015), and as "mathematical graveyards" (Merseth, 2011).

A frequently cited publication, Bailey et al (2010), collectively uses the words "fail," "failed," "failing," and "failure" twenty-eight times. These authors claim that "Developmental education is designed to provide students who enter college with weak academic skills the opportunity to strengthen those skills enough to prepare them for college-level coursework" (p. 255). This claim does not account for the many students whose goals do not include taking a college-level course, ignoring the Bailey et al (2005) finding that 40% of community college students "stated that they wanted job skills or personal enrichment" (p. 8).

Furthermore, Bailey et al (2010)'s conclusion that "fewer than one half of students complete their sequences, and only 20% of those referred to math and 40% of those referred to reading complete a gatekeeper course within three years of initial enrollment" (p. 267), has been cited by other authors without accounting for the many part-time students or the students who never intended to take the course that they identify as a gatekeeper. For example, to promote their alternative curricula, Statway and Quantway, each of the first six reports published by

CFAT use the above Bailey et al. (2010) conclusion without acknowledging students' personal goals (Huang et al., 2016).

Breneman and Haarlow (1998) estimated the cost of all remedial education (not just math) in U.S. public colleges and universities as $1 billion annually, which is "roughly one percent of the institutions' current fund revenues of $115 billion" (p. 2). Unfortunately, when many authors cite the $1 billion cost, they do not include the fact that in terms of the overall budget it is quite small and potentially a good investment in learning.

Confusion Between Correlation and Causation

Large numbers of community college students do place into developmental mathematics; at the same time, many community college students do not graduate within three years. But does this mean that developmental mathematics placement *causes* students to drop out, as many authors have claimed? Or are other factors at play?

Kane et al. (2020), for instance, found that "the role of remedial course requirements as a cause of low completion rates has been overstated" (Kane, et al., 2020, p. 66). Under the Seamless Alignment and Integrated Learning Support (SAILS) program in Tennessee, students who are identified during their junior year of high school as likely to be placed into remedial mathematics in college can enroll in an online class to complete their math remediation during their senior year. Passing the class means being exempt from remediation at any of Tennessee's community colleges. SAILS students did indicate that they felt better prepared for college math. However, after three years of community college enrollment, the SAILS students were not more likely to have completed an associate degree (Kane et al., 2020).

The SAILS alternative did improve students' perceptions of math and of themselves as math learners. However, the program did not significantly impact their college completion plans.

Also, half of the students who were able to take a college-level class as a result of the SAILS program did not pass that class. Although the Tennessee SAILS program did not have the impact on college completion rates that it was intended to have, it did save students who attended college after completing the course, time and tuition money (Kane et al., 2020).

Coordination Between High Schools and College

Calls for better coordination between high school and college mathematics curricula and level of rigor date back well over a century. In 1894, the National Education Association (NEA) Committee of Ten reported on secondary school studies (NEA, 1894). An outcome of this report was a new committee on college entrance requirements. This subsequent report noted that alignment was needed between public high schools and colleges; if not, the gap between high schools and colleges "must be filled by the private schools, patronized by the children of the rich, and the sons and daughters of the great middle class must be deprived of the benefits of a higher education (National Education Association of the United States, 1899, p. 8). Calls for better alignment continue (e.g., Melguizo & Ngo, 2020; Hoyt & Sorenson, 2001).

Reforms and Interventions

Research also includes interventions such as accelerating developmental coursework (e.g., Fong & Visher, 2013; Hodara & Jaggars, 2014) or extending time in developmental math (e.g., Ngo & Kosiewicz, 2017) and alternative curriculum including pathways to statistics and quantitative reasoning (e.g., Cullinane & Treisman, 2010). Many research studies look at the way students are assessed and placed and the impact of placement on student outcomes (e.g., Hu et al 2019; Ngo & Melguizo, 2016; Melguizo et al., 2016; Valentine et al., 2017).

But many of these reforms do not consider reforming teaching or helping the field to better understand what is happening in the developmental mathematics classrooms. Nor do they

help us understand what students are experiencing in these courses. Stigler et al. (2010) found that almost none of the recent reforms have focused on actually changing the teaching methods and routines that define the teaching and learning of mathematics in community colleges. Only within the last five years or so do I find that some studies are beginning to look at what teaching looks like inside the classroom and how students are experiencing it (e.g., Cox, 2015; Cox & Dougherty, 2019).

Policy Shifts

The policy shifts that started in the late 20th century predate the turn toward peer reviewed scholarly research; policy changes continue to be considered and implemented even as we still do not have sufficient research to improve developmental coursework. The authority to set community college policy is primarily held at the local level with few states having the authority to mandate changes at all colleges in their state. However, several states have recently implemented or are considering changes to state laws and policies that impact developmental mathematics enrollment (Visher et al., 2017). See Table 2.1 for a summary of some of these initiatives.

Table 2-1
Recently Proposed or Adopted Policy Changes

State	Recently Proposed or Adopted Policy Changes
Florida	Florida's Senate Bill 1720 exempted the majority of students in Florida's state and community colleges from developmental education; but the bill also required colleges "to offer developmental courses using different instructional modalities, and colleges also had to develop enhanced advising and academic support services" (Hu et al., 2019, p. 3).
Texas	The Texas Higher Education Coordinating Board eliminated the lowest level developmental mathematics course previously offered by community colleges in that state. A new placement test has been adopted and students whose score places them into the eliminated course or below, are not permitted to take the next higher course. Students are to

	be directed away from taking developmental mathematics courses and toward workforce programs. Some of the colleges implemented a bridge course for these students, some allow students to work in a computer lab to increase skill level (Visher, et al, 2017).
Colorado	"Colorado reduced the number of developmental courses in math … and barred colleges from using federal financial aid for student in courses below the high school level." (Visher et al, 2017, p. 5-6). As of 2015, most of Colorado's community colleges had adopted online tutorial packages, placement-test preparatory classes, or referral to agencies such as adult basic education programs or workforce development offices. (Visher et al, 2017).
Connecticut	Connecticut introduced a reform in 2012 that places highest-scoring students in corequisite courses; middle-scoring students were provided with a one semester accelerated developmental course; the colleges were expected to develop preenrollment courses for the lowest-scoring students (Visher et al., 2012).
Missouri	Missouri House Bill 1042 requires all of its public two-year as well as four-year higher education institutions to replicate best practices to be defined by a task force. Recommended reforms include the establishment of minimum academic competency levels for enrollment in developmental courses. (Visher et al., 2012).

Situating My Study in the Research Literature

Sitomer et al. (2012) propose a four-strand research agenda that connects to AMATYC's strategic plan for research in community college mathematics education. These strands are in the areas of community college mathematics instruction, students, curriculum, and technology and e-learning. My work lies within the second strand which summarizes the need for research on students: "It is essential to understand the students who are taking our classes in order to capitalize on their strengths and provide adequate support to overcome the challenges they encounter as they progress through mathematics courses" (Sitomer et al., 2012, p. 36).

I argue that we cannot improve students' experiences and outcomes in developmental mathematics unless we gain a deeper, more complex understanding of who these students are as

math learners. These students bring strong emotions and beliefs about mathematics and beliefs about themselves as learners that they have developed over many years before arriving at college. Thus, attending closely to these students' voices about their experiences is critical to the study and improvement of developmental mathematics courses. To begin unpacking students' experiences, I turn next to the literature on mathematical affect in the math education literature, before I describe my conceptual framework for this dissertation.

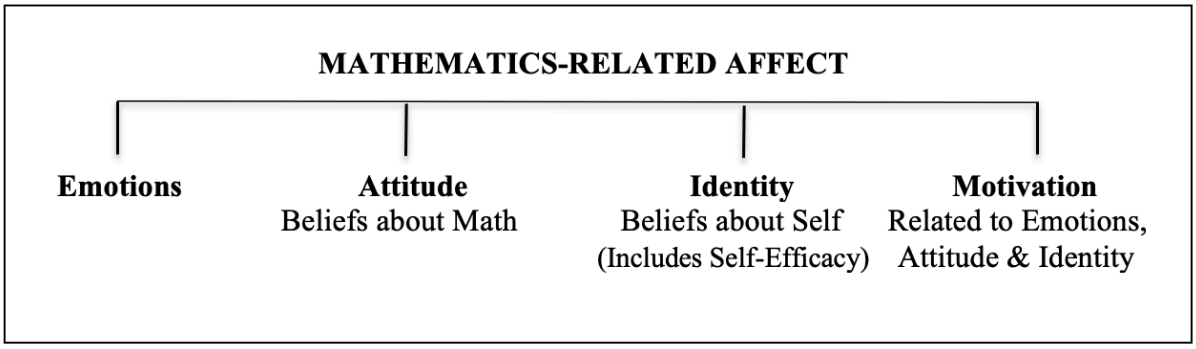
Affect in Math Education

Mathematical thinking is significantly influenced by affective features. The word "affect" is often used "as an overarching umbrella concept that covers attitudes, beliefs, motivation, emotions, and all other noncognitive aspects of "human mind," but it has also been used to refer to emotional states and traits, more narrowly; (Hannula, 2020, p. 32). One of the topical surveys to come out of the 13th International Congress on Mathematical Education (ICME-13) was an overview and summary of the state of research on attitudes, beliefs, motivation and identity in mathematics education, collectively referred to as mathematics-related affect. These authors examined significant strands of research and found the following broad categories of affect: emotions, beliefs about math, beliefs about self, and motivation (Hannula et al., 2016). I begin this section with a brief definition and discussion of the significance of each category identified by Hannula et al. (2016), before I discuss the narrative-identity framework of Sfard and Prusak (2005). Finally, I describe my own conceptual framework, which centers narrative-identity, while also considering other areas of mathematics-related affect as well as life and learning experiences that relate to students' mathematics learning.

Major Categories of Affect in Math Education

Figure 2.5 illustrates the broad categories of mathematics-related affect as described by Hannula et al. (2016) that are relevant to my study.

Figure 2.5
Major Categories of Mathematics-Related Affect



Note. Figure 2.5 is based on the ICME-13 Topical survey on mathematics-related affect (Hannula et al., 2016).

Emotions

Any mathematical task with which a student is engaged involves emotions. Emotions are an important part of human memories and play a role in coping, influence cognitive processing, direct attention and "will influence the choice of strategies in the future" (Hannula, 2020, p. 33). As math learners begin to engage in mathematical tasks, emotions such as elation, anxiety and curiosity help to focus their attention; they may also feel anticipatory emotions such as hope or anxiety. Self-reflection on success or failure may lead to emotional responses such as pride, boredom, anger, frustration or apathy as described in Figure 2.6 (Hannula et al., 2016, p. 2, 22). Understanding students' emotions is key to understanding their motivation to put effort into learning mathematics.

Figure 2.6
Emotional Responses to Self-Reflection of Success or Failure

PRIDE Emotional response to success in a high value task	**BOREDOM** Emotional response to success in a low value task
ANGER / FRUSTRATION Emotional response to failure in a high-value task	**APATHY** Emotional response to failure in a low-value task

Note: Figure 2.6 is based on the findings of ICME-13 Topical Study (Hannula et al., 2016)

Attitude

Hannula et al. (2016) found that a variety of definitions for attitude have been used by mathematics education researchers. Two definitions in particular are most recurrent: (1) attitude as feelings associated with math as positive or negative and (2) attitude as emotion, belief and behavior toward mathematics. The second definition appears to contradict the ways that the survey uses the term affect. This is an international study and when I checked some of the referenced articles, it appeared to be an issue with translation. The authors developed a working definition of attitude as a function of the problem posed by a researcher. In other words, they argue that attitude is a construction of the observer, rather than the individual. For example, they point out that a teacher's claim that a student has a negative attitude may be an indication that the teacher is giving up on the student rather than an accurate interpretation of the student that can be acted upon. This working definition is important because it positions the student as an authority on their feelings toward mathematics that underlie their beliefs and motivate their behavior, rather than accepting observers' interpretations at face value.

As a consequence of this definition, these authors question the use of attitude scales, where the respondents express the extent to which they agree or disagree with pre-selected items that have been chosen by others. They point out that items chosen by researchers may be irrelevant to the respondents. Instead, they propose that narrative approaches to research on attitude are better able to reflect what respondents consider relevant about their relationships with mathematics (Hannula et al., 2016).

Identity

This ICME-13 group also found that prior to the work of Sfard and Prusak (2005), studies on identity did not provide a clear definition of how the term was being used. Most studies frame identity as personal but at the same time treat identities as constructed in relationships with others. Literature on student identity also links race, gender and ethnicity with issues of identity. Nearly all studies Hannula et al. (2016) reviewed on identity are related to socio-cultural theories of learning, rather than psychological theories of identity development. Theoretical links have been made between identity and mathematical learning. The focus of many of these studies is on participation, which considers how identity develops based on a feeling of being included in or excluded from the mathematics learning community.

Self-Efficacy. The ICME-13 authors found that self-efficacy is an important part of one's identity and may change from year-to-year according to changes in the classroom environment. In this view self-efficacy is a self-evaluation that is situation dependent. They also found that Bandura's (1977) definition of self-efficacy as an individual's belief in their own ability to put into effect a course of action that will result in a desired attainment continues to be widely accepted today (Hannula, et al., 2016).

Further, these authors found that students' self-efficacy beliefs about their ability to carry out a task are created and changed as they interpret four types of experiences: mastery, vicarious, verbal persuasion, and psychological reaction. Mastery experiences, based on a person's past successes or failures, have the greatest influence on a person's self-efficacy. Vicarious self-assessment is based on one's comparison to others. Verbal persuasion by others has limited impact. Students may be misjudged as incapable when they experience a psychological reaction to feeling tired or stressed.

Although studies show that self-efficacy and mathematics achievement are related, there is a lack of empirical support for causal ordering. In other words, it is not clear whether self-efficacy leads to achievement or achievement leads to self-efficacy. Recent research indicates that there is a reciprocal relationship between self-efficacy and achievement and that the dominant effect is from achievement to self-efficacy (Hannula et al., 2016). This is important because it demonstrates, for example, that simply telling someone that they can learn something is not likely to convince them if past performance has not resulted in meaningful learning.

Motivation

Hannula et al (2016) define motivation as "the reason we engage in any pursuit, mathematical or otherwise" (p. 18). Students have goals that help them decide how much effort to put toward mathematical activities. They use cognitive as well as affective resources to help them achieve their goals.

Interest is one of the most significant motivational factors and one of the strongest predictors of achievement and persistence over time. It also helps to predict course taking, long-term achievement and mathematical identity. Students also tend to be motivated by the degree to which they feel their efforts will eventually help them achieve a non-mathematical long-term life

goal. Identifying specific and having a plan for achieving goals can lead to more productive engagement with mathematics. A strong sense of self-efficacy from engaging in tasks related to their goals, may fluctuate from year to year and is related to help-seeking behavior (Hannula et al., 2016).

Sfard and Prusak's Narrative Identity Framework

A central piece of my conceptual framework, described below, is the narrative-identity framework developed by Sfard and Prusak (2005). According to these authors, development of an identity enables us to "*cope with new situations in terms of our past experience and gives us tools to plan for the future*" (p. 16, emphasis original). Identity may provide comfort to students, based on the expectation that what one experiences now, will happen again in a similar situation in the future (p. 16). The authors developed this framework to help answer the questions: "*Why do different individuals act differently in the same situation? And why, differences notwithstanding, do different individuals' actions often reveal a distinct family resemblance?*" (p. 14, emphasis original).

These authors argue that identity is human-made, not inborn, and can change over time as we receive messages from others as well as institutional messages in the form of grades, test scores, diplomas, and diagnoses, to name a few. According to this framework, a person's identity is the stories they tell in the form of words that are taken seriously and shape one's actions (Sfard & Prusak, 2005). It is not enough to define identity as "who one is;" these authors define identity as "collections of stories about persons or, more specifically, as those narratives about individuals that are *reifying, endorsable*, and *significant*" (p. 16, emphasis original). In other words, identity narratives are a collection of stories students tell about themselves that are concrete, reflect the state of their world, and imply membership or inclusion in the mathematical

community. Changes to these stories would change how they feel about themselves (Sfard & Prusak, 2005).

Sfard and Prusak's (2005) framework is significant because it highlights that what students bring with them to instruction (i.e., prior experiences that have shaped their identities), may be as important to their learning as the instruction they experience in the present. The ubiquity and repetitiveness of messages students have received can impact the way that they interpret and endorse new messages. This, in turn, can lead to decisions (consciously or unconsciously) about engagement (e.g., completing homework assignments) that *appear* irrational to present observers but that make a great deal of sense within the context of the students' prior experiences.

Sfard and Prusak (2005) also assert that a potential source of one's identity is a designated identity that comes from stories about others that a student may relate to such as parents, or heroes. These stories may lead one to feel that they have been assigned similar identities. For example, a parent who says that they were never good at math may lead their child to feel that the same fate awaits them. According to the authors: "Changing designated identities that have been formed in childhood is a particularly difficult task" (p. 18). Designated identities are not necessarily a matter of choice, but they are not entirely immutable; they are created by narratives and are not limited to a single author. Designated identities have the potential to act as self-fulfilling prophesies and "are likely to play a critical role in determining whether the process of learning will end with what counts as success or with what is regarded as failure" (p. 19).

However, identity can change. Learning has the potential to help lessen the impact of designated identities and open spaces for students to begin to develop new identities with respect to mathematics (Sfard & Prusak, 2005). In order to design the kinds of courses and instructional

experiences that can shift developmental mathematics learners' identities in ways that will support their success, instructors need to better understand the mathematical experiences and resulting identities that students bring with them to their coursework. In the next section, I will unpack my conceptual framework for investigating aspects of mathematics-related affect and experiences.

Conceptual Framework

The ICME-13 topical survey (Hannula, et al., 2016) had not yet become publicly available when I started this study. My initial conceptual framework focused on the narrative-identity framework of Sfard and Prusak (2005). I used this framework together with a set of analytic questions that I detail in Chapter 3 as well as my research question to design my data collection and to guide my initial data analysis. But after doing an initial data analysis, I realized that there were some important features of participants' experiences that were not being captured well by my codes. Most notable was the role of emotions in the ways that my participants experienced learning mathematics. When I asked questions related to identity, my participants often described their emotions, as well. So, I searched the literature on mathematics-related affect, found the ICME-13 topical survey described above, and redeveloped my conceptual framework.

The ICME-13 authors, Hannula et al. (2016) noted that:

> Though the concept of identity seems to function as a nexus of social narratives and subjective experience, most literature reviewed here on both student and teacher identity is quite detached from studies dealing with emotions (e.g., mathematics anxiety), attitudes, or beliefs.

I found this in the literature as well. These authors recommended future study of the intersections between identity and other aspects of affect. I did not set-out to find these connections, but I did find them as I will describe in Chapters 4 and 5. Further, I found that the relations between

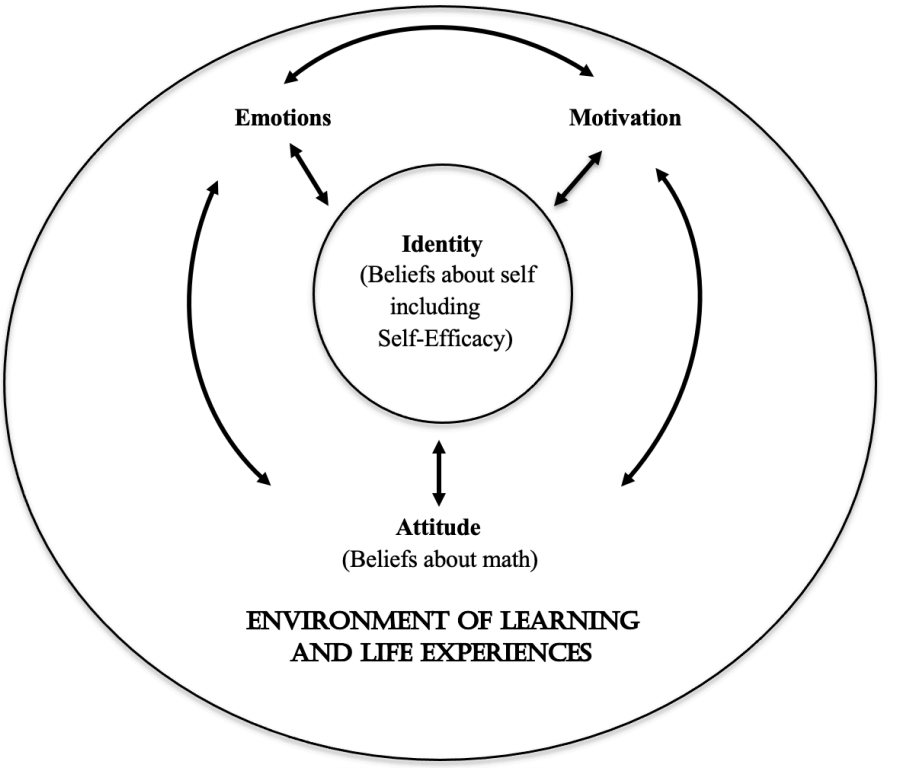
different aspects of affect were reciprocal for my participants. The reciprocity among aspects of affect is reflected in my conceptual framework as described below.

Sfard and Prusak's (2005) narrative-identity framework is central to the conceptual framework that guided my analysis of students' accounts of their past learning and life experiences. But when I asked my participants about their identity, they also told me about their emotions such as anxiety, joy and gratitude, often without prompting. So, I include emotions in my framework as well. Additionally, as noted above, the ICME-13 authors found that emotions influence one's cognitive processing and one's focus on mathematical tasks. Although self-efficacy is sometimes expressed as distinct from identity, it is an important part of a student's identity as a math learner and is strongly influenced by their past experiences while engaging with mathematics, so I place self-efficacy along with identity at the center of my framework. Identity and self-efficacy impact students' motivation to put forth effort, but students' attitudes, defined as beliefs about mathematics, may influence their behavior as well. Attitudes may also be influenced by their emotions.

My conceptual framework is illustrated in Figure 2.7. Narrative-identity, which includes self-efficacy, impacts and is impacted by emotions, attitude and motivation. There is also a reciprocal relationship among emotions, attitude and motivation. Each of these aspects of mathematics-related affect are experienced within the environment of learning and life experiences.

Figure 2.7

My Conceptual Framework: Identity is Central to Other Aspects of Mathematics-Related Affect in the Context of the Environment of Learning and Life Experiences



Mathematical thinking is influenced by each aspect of mathematics-related affect. Developmental mathematics students' prior learning and life experiences, both in and out of the classroom, are varied in nature. These experiences have the potential to support or hinder meaningful learning of mathematics. Their identities and other aspects of mathematics-related affect can impact their ability to recognize and take advantage of these experiences.

Chapter 3
Methods of Data Collection, Analysis, and Presentation

The purpose of my research study is to develop an understanding of who developmental mathematics students are as math learners by examining their past learning and life experiences, with a particular focus on their mathematics-related affective characteristics of identity, emotion, attitude and motivation. I explored the memories of a group of 10 developmental mathematics students to learn about possible characteristics and past experiences developmental mathematics students might bring with them to college. I used my conceptual framework as a lens to consider the identities and associated affective characteristics that these students developed as they experienced life and learning. In this chapter, I detail the methods I used to answer my research question: *What do developmental students' accounts of their past learning and life experiences reveal about them as math learners that their grades do not?* I describe portraiture as my method of analysis and data presentation and explain why I choose this method. I also report on my research design, participant recruitment and participation, the data I collected from my participants and how I collected that data. I explain how and why I chose four focal participants for in-depth portraits. Finally, I consider what Lawrence-Lightfoot and Davis (1997) refer to as "authenticity," including a statement of my researcher positionality which these authors refer to as "voice of autobiography," to position myself with respect to this project and my findings.

Portraiture as Method of Analysis and Data Presentation

Portraiture is a qualitative research methodology developed by Lawrence-Lightfoot to highlight the complexity and subtlety of human experiences within the socio-cultural context of

those experiences. As a portraitist, the researcher not only records, but also interprets the experiences and perspectives of the study participants while acknowledging the wisdom and authority they bring to the study (Lawrence-Lightfoot & Davis, 1997).

The inspiration for this methodology came from Lawrence-Lightfoot's experiences as the subject of artists' portraits on two different occasions. Through these experiences, she found that through the human interaction between the artist and subject, portraits can capture the essence and spirit of the subject. As a research methodology, the researcher may discover things that participants had not realized themselves and may show participants in a way that they have not been seen before. The subject is to be viewed as a person, not an object. Just as different observers of a drawn portrait have different interpretations, readers of a researcher's portrait will have different interpretations as well. The researcher should provide enough detail so that individual readers can make their own interpretations, though it is expected that these interpretations will have common resemblance (Lawrence-Lightfoot, 1983).

Portraiture is "framed by" phenomenology in that both methodologies seek to illuminate the complexity of human experience (Lawrence-Lightfoot & Davis, 1997). Traditional experimental research that looks for generalizable knowledge by varying and controlling the conditions of particular interventions may overlook the possibility that a given intervention may have different effects on different individuals as the result of their sociocultural conditions. Phenomenology, by contrast, does not try to "explain and/or control the world, but rather it offers us the possibility of plausible insights that bring us in more direct contact with the world" (van Manen, 2016, p. 9). For example, students in developmental mathematics classes come from different high schools and different backgrounds; they also vary in age and life experiences. Their prior learning and life experiences will likely impact the ways that they experience

instruction and the expectations of college. Phenomenology offers a way to account for this context by attending specifically to how it shapes phenomena of interest.

At the same time, phenomenologists do look at what participants have in common and attempt to describe a universal essence of a given phenomenon by developing coding schemes common across all participants in a study (Bloomberg & Volpe, 2018, p. 54, 96). Phenomenologists investigate the way "we" experience a phenomenon (van Manen, 2016, p. 9). Portraitists, on the other hand, delve deeply into unique individual experiences of phenomena. Their goal is to understand the complexities of the individuals.

When designing this study, portraiture first resonated with me as a potential methodology because I often find that my view of developmental mathematics students is different from what is typically presented in the literature. Given my research question in this study, portraiture allows me to look at collective as well as individual experiences throughout participants' years of learning mathematics, with a focus on the uniqueness of the individual. Developing a separate coding scheme for each individual participant enables me to seek out the complexity and uniqueness of their prior experiences and identity formation. In this way, I can work to understand what the participants view as central issues with "no preconceived notions of key themes" (Lawrence-Lightfoot, 1983, p. 14). Generalization to the general population is not the goal (Lawrence-Lightfoot, 1983; Lawrence-Lightfoot & Davis, 1997). My goal was to examine and understand how some developmental mathematics students experienced life and learning prior to enrollment in developmental mathematics courses and to understand the affective characteristics they bring with them to college.

Essential Features

Lawrence-Lightfoot and Davis (1997) describe five features that they consider essential to portraiture methodology: context, voice, relationship, emergent themes, and aesthetic whole. These features work together to create the portrait.

Context

The context in which the action takes place is considered to be a resource for understanding what the participants say and do by helping the researcher to determine what details are most relevant and important to include and what to leave out. The context primes the canvas and usually opens the portrait to help the reader see the whole person by situating their experiences (Lawrence-Lightfoot & Davis, 1997). Because I am examining my participants' past learning and life experiences, the context is different for each participant. In order to more fully understand their experiences, it was crucial to do what I could to capture the contexts of those experiences. Context is brought into the portrait in five ways.

Internal Context. Lawrence-Lightfoot and Davis (1997) refer to the physical setting as the internal context because it helps to provide a backdrop to the portrait and recognize the importance of the environment in which actions take place. The physical setting of the community college is less relevant to my study because I am not investigating my participants' experiences there. However, the physical setting in which prior learning experiences occurred may be relevant for different participants. For example, my participant Brad described a prison-like atmosphere in a predominantly Black high school that included classrooms with no windows as part of his description of the ways that students were poorly treated there.

Personal Context. The influence of the researcher's past must be acknowledged and made explicit: "The portraitist is clear: from where I sit, this is what I see; these are the

perspectives and biases I bring; this is the scene I select; this is how people seem to be responding to my presence" (Lawrence-Lightfoot & Davis, 1997, p. 50). The researcher's prior research and experiences generate theoretical expectations and influence the researcher as well as the portrait. I describe my researcher subjectivity, which Lawrence-Lightfoot and Davis refer to as authenticity, at the end of this chapter.

Historical Context. This refers to the history, culture and values of the institutions at which actions, such as learning, take place. The very essence of my study concerns participants' histories as math learners and how those histories have been shaped by their in-school as well as out-of-school experiences. I rely on my participants to describe the aspects of prior institutions that are relevant to them.

Aesthetic Features. Portraits should not be written in strictly academic language; they should be understandable to those beyond the academy. When writing portraits, the portraitist should strive to write with elegance and in a manner that will result in a portrait that is pleasing to the reader. Symbols and metaphor may help capture the readers' attention and compel them to read further. For example, I have always felt that many of my developmental mathematics students are like canaries in the coal mine who can alert us to the often unrecognized privileged that shapes learning experiences that many instructors, administrators, policy makers and researchers have had which may include a privileged relationship with mathematics.

Shaping Context. The portraitist must be aware that while actors are shaped by the context, they also give it shape (Lawrence-Lightfoot & Davis, 1997). The ways that students react to their environment has an impact on the environment itself. My participants were shaped by the messages they received which influenced their actions and their actions (or reactions) also shape the messages they receive.

Voice

All researchers have a point of view. Even in quantitative research, where the researcher's voice is presented as purposely silenced, researchers' perspectives always inform the work because researchers select "the research question, design of the study, data collection strategies and … interpretation of data" (Lawrence-Lightfoot & Davis, 1997, p. 86). The use of first person in portraiture is intended to highlight, rather than hide, the researcher's presence. Lawrence-Lightfoot & Davis (1997) describe six ways that voice may be used in developing a portrait.

Voice as Witness. In this case, the researcher's position is distanced enough to see the whole. The portraitist may notice patterns, view the participant's experiences through new eyes, picking up on things that may have become so familiar by the participant to be unnoticed.

Voice as Interpretation. All researchers interpret data to make sense of it. The portraitist's goal is to present their interpretation to improve understanding, while providing enough detail for the reader to be able to make their own interpretation.

Voice as Preoccupation. Portraitist's bring a conceptual framework based on their own prior work and their reading of the literature. This preoccupation can be purposefully developed to guide the study under investigation and shape the portraitist's interpretation. One must be critical of the literature that forms the basis of this preoccupation.

Voice as Autobiography. The portraitist's point of view is also shaped by their own life experiences. The knowledge and wisdom that come from these life experiences are seen as resources for the portraitist to identify and connect with the participants while being careful to maintain the participant's intentions in the retelling of their story.

Voice Discerning Other Voices (Listening for Voice). In portraiture, the portraitist co-constructs the participant's story with them, listening closely for the participant's own meaning and interpretation of their words.

Voice in Dialogue. The voices of the participant and portraitist come together in their dialogue and influence questions asked and answered during the interview. Though the portraitist must ensure that their voice does not overshadow that of the participant, their voice is nonetheless a vital component of the dialogue (Lawrence-Lightfoot & Davis, 1997).

Relationship

Relationships are central to portraiture. The portraitist benefits from developing a close relationship with participants in which the portraitist intentionally builds trust and rapport. These relationships help to provide the participant with a safe space to express feelings of vulnerability, anxiety, and prejudice. At the same time, the portraitist must always be aware of her responsibility to protect the participant from potential harm (Lawrence-Lightfoot and Davis, 1997). Portraitists build trust and rapport in relationships in three ways.

The Search for Goodness. According to Lawrence-Lightfoot and Davis (1997), social scientists have a tendency to focus on what is wrong, looking for remedies. Portraiture, by contrast, looks for greater balance including a focus on strength and goodness. This helps to build relationships in which participants feel they can express their feelings without being judged.

Empathetic Regard. The portraitist tries to understand the participant's experiences and actions from the participant's perspective, building a strong relationship through empathy.

Reciprocity and Boundaries. Boundaries are important for protecting the vulnerabilities of the participant. As such, boundaries serve as a counterpoint to the development of an intimate

relationship. Boundaries are an ethical responsibility of the portraitist. The portraitist also protects the validity of the research by staying focused on the research question and not probing beyond what the participant is comfortable sharing.

Emergent Themes

Lawrence-Lightfoot and Davis (1997) describe emergent themes in portraiture as analogous to pattern level codes used by other qualitative researchers. The conceptual framework that guides a portraiture study is a reflection of the researcher's views based on reviewing appropriate literature and prior experience. This framework, together with a set of analytic questions developed prior to analysis, provide guidance as the portraitist searches the data for patterns. These thematic patterns are used to construct the portrait in an iterative manner. The portraitist searches for and constructs themes in five ways.

Repetitive Refrain. Participants may describe a particular type of experience, but similar experiences may be expressed in different ways which are sometimes easily recognized or may be subtle, requiring careful attention to detail. The portraitist, therefore, tries to determine the meanings behind the participant's expressions and stories. Coding can bring together pieces of a theme that has been expressed in different ways at different times.

Resonant Metaphors. Some themes may emerge from the metaphors that participants use to relay their experiences. These themes may be difficult to detect; however, they are identifiable by the ways that seem to resonate for the participant. The portraitist must listen carefully to identify these themes when they appear and to parse their meaning to the participant.

Cultural and Institutional Rituals. Stories involving rituals surrounding life's milestones may be of particular significance to a participant. The portraitist can use these themes to better understand what is important to the participant.

Triangulation. The portraitist may look for points of convergence among various types of data collection. In my study, I looked for internal consistency between the transcript, profile sketch and background questionnaire. I also looked closely within each interview transcript for discrepancies. Sometimes what initially appeared to be a discrepancy provided me with greater insight into a students' perspective.

Revealing patterns. Sometimes stories are scattered, and patterns are not clear. Dissonant views may emerge that run counter to the bulk of a participant's experiences. In portraiture, such dissonant views are not considered outliers to be dismissed but are to be understood and explained in ways that add greater depth of interpretation. The connections the portraitist makes may not be apparent to the participants, as they may be too close to the stories. However, these connections are important for understanding the complexities of the participants' experiences.

Aesthetic Whole

The themes provide a structure for the portrait as the portraitist brings them together into a complex whole. The portraitist also looks across themes to express them with attention to aesthetics. Order and logical relationship among them help the reader to see patterns more clearly. Finally, although a dominant theme may have been found, the portraitist looks for a comprehensive view of the themes. It may be the case that some things do not seem to fit; these should be reconsidered in light of the overall portrait to determine if they contradict or support the dominant theme.

Data Collection

I wanted to collect and analyze data that would allow me to look beyond knowing the numbers of students who indicated on a survey that they either do or do not like math, do or do

not think it is important, or believe they can learn. Rather, I wanted to investigate the complexities of how their mathematics-related affect developed in relationship with their past experiences to gain a more complete picture of who my participants are as math learners. I also wanted to understand the chances they had had to learn mathematics in a meaningful way prior to enrollment in college coursework. This is information that can come only from students themselves. In this section, I describe the population of students from which I recruited participants and the data I collected from those participants. I also describe how I chose four focal participants from among the larger group of 10 participants for in-depth portraits.

Research Participants

Although both two-year and four-year colleges offer developmental mathematics courses, most students who take these courses do so at a community college, so it makes sense to study students there. As I described in the literature review, in this study I use the term "developmental mathematics" to refer Pre-algebra and Elementary Algebra courses. I decided to limit study participants to students who enrolled in one of those two courses at a community college. I had hoped to also limit recruitment to Pre-algebra, but later expanded my recruitment to include two students whose initial enrollment was Elementary Algebra, as I explain below. I recruited without regard to any definition of "success" and included students who passed, failed, or withdrew from their courses. Instead, I asked participants for *their* definitions of success.

In keeping with my research goals and portraiture methodology, I wanted to recruit some of my own former students so that I could build on prior relationships and gather rich data. I initially recruited students from two of my prior Pre-Algebra classes. However, I felt that the sample was not diverse enough by race and gender, so I expanded my recruitment to include two additional students whose initial enrollment was Elementary Algebra. I also recruited from two

Pre-algebra classes taught by other instructors to learn more about the experiences of students whom I had not previously taught. In all, my sample comprised 10 participants total, eight of whom were my former students. I purposefully choose four focal participants for in-depth portraits using my conceptual framework. I describe this process later in this chapter.

All participants were enrolled in at a suburban community college in the Midwest during at least one semester from the fall of 2017 through the fall of 2019. Several participants shared highly personal and confidential information. So, although participants were asked to provide a pseudonym for the study, as an extra layer of identity protection, I do not specify the particular semester(s) in which individual students were enrolled within that three year time frame. In addition, I only specify if their initial course enrollment was Pre-Algebra or Elementary Algebra if it is relevant to my findings.

I did not approach any of my former students until at least two months after the end of the semester in which they were enrolled in my course. Furthermore, as the focus of my study is on students' experiences, identity formation and related affect *leading up to* initial enrollment, I did not include my in-class observations of my students. Although I did not ask students about anything related to the class that they took with me, participants sometimes brought up things from our time together. Out of respect, I allowed them to finish their thought before moving on, but I do not generally include that information in my findings. If a student described passing or not passing my class before moving on, I may include that because it is relevant when I compare my participants' definitions of success with that of the literature. There is a line item on the consent form that asks my former students for permission for me to include their grades and "pre-tests." However, after going through several iterations of my study, I found that this information was not useful in answering my research question and I decided not to use it.

Data Collection Documents

My data collection documents consisted of a background questionnaire, a profile sketch, and protocol for semi-structured interviews. Prior to contacting students for participation, I submitted a University of Michigan IRB (Internal Review Board) application which included informed consent forms as well as the data collection documents, and I received an exemption. Each time I updated a data collection document, for example, after the pilot interview, I submitted the revised document for IRB approval and each time my exemption was upheld. I also sent these documents to and received permission from the Research Review Board at the school at which I collected my data. I will only refer to as Midwestern Community College. All interviews took place on the college campus. These data collection documents and informed consent forms are in the appendix:

A1: Informed consent form (for my former students)

A2: Informed consent form B (not my students)

A3: Background Questionnaire

A4: Interview Protocol

A5: Profile Sketch

Background Questionnaire

I developed a background questionnaire which includes questions about the types of mathematics courses participants took and outside time commitments that they had during high school. I asked how prepared they felt for college math as well as a chronology of college math classes that they had taken. In addition, I included some very basic math problems with questions about how well participants felt they understood the problems. Demographic questions include age, gender, race, whether participants speak English as a first language, whether participants

receive financial aid for college, whether they are first in their family to go to college, disability status and accommodations, as well as questions about discrimination and microaggressions.

I revised the questionnaire several times. The most significant revisions concerned discrimination and micro-aggressions. Although I do specify in the consent form that I would be asking about race, my initial participants seemed surprised by these questions. Because of this, I added two questions to the questionnaire. I first asked if they had at any time experienced racism, or discrimination, including microaggressions. I asked participants to reflect on discrimination and microaggressions related to gender, social class, disability or any other identities in addition to race and asked them to explain what they experienced. I also asked if they felt their opportunities to learn had been impacted based on any of these identities. I followed up on these answers during the interview.

I asked participants to complete the background questionnaire and send it back to me prior to the interview so that I could have time to look it over. I explained that they had the option of bringing the already filled out questionnaire with them to the interview but that if they chose to bring it with them, I would need about 20 minutes to review it before the interview began.

The background questionnaire was helpful in two ways. First, it gave participants an idea of what we would be talking about and allowed them to think through their written responses and to consider the extent to which they wanted to share sensitive information. Second, it helped me to make more efficient use of interview time by highlighting potential areas for greater focus. I did not use the questionnaire to narrow down the pool of participants or to select focal participants. All participants who provided a questionnaire also participated in an interview.

Interviews and Interview Protocol

All interviews took place on the campus of the community college in which the participants had enrolled in a developmental mathematics course and lasted approximately one hour. All interviews were audio recorded and transcribed. I also followed-up on some of their answers on the background questionnaire. My goal was to learn about what the participants thought was important to know about their past learning and life experiences. Rather than adhering to a rigid protocol, I allowed them to provide details that were important to them.

During each interview, I used a profile sketch to explore participants' perceptions of themselves as learners of math (Gholson & Robinson, 2019). This sketch is shown in Appendix A5. I introduced the sketch by explaining to each participant that there are stories we tell ourselves about who we are as a learner of math. We also hear stories and receive messages about ourselves from other people. These can be things someone tells us directly or can be messages we receive indirectly such as feedback on tests or things we overhear people say. I then asked participants to write about messages they received from others in-school as well as out-of-school on the outside of a head and shoulders profile; on the inside, they were asked to write about messages they told themselves about who they are as a math learner. I asked participants to complete these profile sketches several times because messages may change during different periods of learning throughout K-12 and beyond. The profile sketches seemed to prompt participants to think through perceptions they have of themselves as doers and learners of math and to think about the messages they received.

I began each interview by asking participants to think back to their earliest memories of learning math and to tell me about what they remember. This question served as a warm-up to the first profile sketch. After participants completed each profile sketch, we discussed their

entries. (See Appendix A4 for further details). My intention was to have three profile sketches, one for their elementary school years, one for their middle school years, and one for their high school years. But because of their individual experiences, this varied. For example, one participant who was homeschooled had her mother as her teacher and her siblings as classmates throughout K-12. She felt strongly that messages were consistent and that additional sketches would not be any different.

I approached each interview with an awareness of my positionality, which I describe below, as well as an awareness that compassion and understanding are essential parts of my responsibility as a researcher. I followed participants' lead at all times. There are places in the portraits where the reader might feel that some information is missing. To protect the anonymity of my participants, there are places where I felt that specific details could be too revealing. Also, there were a couple of instances where I did not follow-up on my participants' response. If I had noticed by their words, tone, or body language that they were experiencing some level of discomfort, I moved on to another question. Out of an abundance of caution, I backed off, and did not push participants to talk about things if I sensed it might cause pain or discomfort. What is personal or potentially painful to one person, may not be to another.

Data Analysis

Coding Process

The revised conceptual framework as shown in Figure 2.7 was not part of my initial coding because I had not yet developed it. Instead, I began coding by using Sfard and Prusak's (2005) narrative-identity framework and my research and analytic questions to closely examine the data and develop an index of the common and/or important topics, details and ideas each participant discussed. After coding each of the transcripts and reviewing these initial codes, I

developed an appreciation for the role of emotions in learning mathematics. I noticed that it was often the case that when I asked a participant about their identity they also talked about their emotions. So, I searched for literature on mathematics-related affect for a better understanding of how the field situates identity with other categories of affect, including emotion. It was at about that time that the ICME-13 topical survey on mathematics-related affect became available. At this point, I revised my conceptual framework to incorporate each aspect of affect along with identity and experiences.

Initial Coding

The narrative-identity framework of Sfard and Prusak (2005) guided my initial data analysis along with my research question and a set of previously developed analytic questions. These questions reflected what I hoped to learn about my participants' prior experiences based on my teaching experiences and pilot study. These analytic questions were:

(a) What identities did a sample of DM students develop as math learners prior to their enrollment in DM as reflected in their narratives?

(b) How did these identities shape their actions as math learners prior to enrollment in DM?

(c) What opportunities and barriers to learning did these DM students report experiencing, in school and out-of-school, prior to enrollment in DM?

(d) In their perspectives, how was their learning of math in K-12 shaped by such learning barriers and opportunities? How important did they consider this to be?

(e) What did these students say is hard about learning mathematics?

(f) How did they approach basic mathematics problems?

(g) How did these students define success in mathematics?

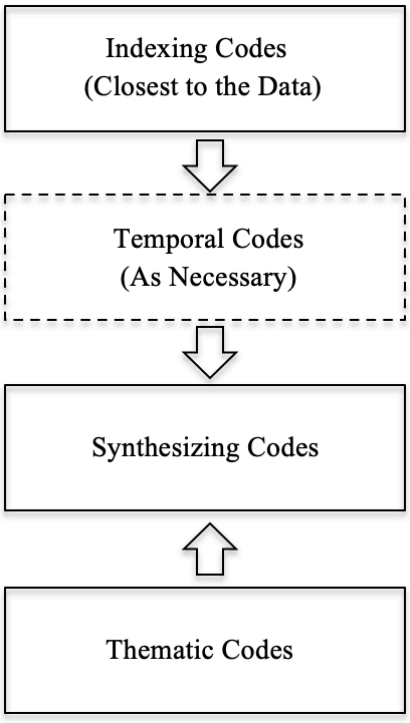
With these analytic questions as well as my research question and identity framework, I examined each individual transcript and began coding, line-by-line, very close to the data. My goal was to condense the data while capturing the participants' meaning, as well as using in vivo coding (Miles & Huberman, 1994) to capture what seemed most meaningful to the individual participant. I refer to this level of coding as *indexing*. I iterated my sets of indexing codes several times for each participant to help ensure that I was capturing the essence of what they were saying.

Next, I returned to my questions and the narrative-identity framework to develop top-level, or *thematic* codes, for each individual participant. I looked across my indexing codes to identify connections among the topics, details, and ideas a participant raised that spoke to my questions and/or narrative-identity framework. For example, for one participant I had indexing codes to capture the *hate-filled atmosphere* and *mistreatment of students* he experienced while attending a predominantly Black high school. I developed the top-level code *Learning Environment* to group these indexing codes.

To aid my search for themes, I also developed *synthesizing* codes to group related indexing codes together in ways that informed and were informed by the emerging thematic codes. For the example, under the thematic code *Messages*, some synthesizing codes included *In-School*, *Out-of-School,* and *Self-assessment* to group indexing codes related to identity messages. If an indexing code indicated changes in a participant's experience over time, I used *temporal* codes to group the indexing codes within the synthesizing codes. For instance, one participant, who uses the pronouns they and them, described a significant change in identity messages they received starting in 7th grade and another significant change that started in 9th grade. So, I grouped these messages using the temporal codes *Early Years*, *Middle Years,* and *High School*

and Initial Postsecondary under each synthesizing code under Messages to capture the change over time of the messages they received. Just as with the indexing codes, I iterated the synthesizing and thematic codes for each participant many times in order to help ensure that I was capturing the essence of what they were saying. Figure 3.1 offers a simplified illustration of my coding process.

Figure 3.1: *Development of Code Categories*



Note. This was a very iterative process. The arrows indicate the general direction in which coding proceeded.

Re-Coding

As described above, I developed my conceptual framework (see Figure 2.7) after initially coding all of the transcripts. After my initial data analysis, I realized that there were some important features of participants' experiences were not being accounted for. Most significant was the way that emotions impacted and were impacted by my participants' experiences. So, I

returned to the literature on mathematics-related affect, as I described earlier in Chapter 2, and redeveloped my framework. I situated Sfard and Prusak's (2005) narrative-identity framework in the center of emotions, attitude and motivation. I then used this new framework, along with my existing analytic questions, to reconsider all of the codes I had developed for each participant. At this point, I also coded the profile sketches and background questionnaires for each participant.

Importance of Individual Coding

When coding, I focused on the data for each participant individually because my goal was to deeply investigate their individual experiences. My participants came from a range family backgrounds and prior school districts; they also varied by age, race, gender and other demographics. Portraiture methodology allowed me to look deeply into the past learning and life experiences of each participant to gain meaningful insights into how their past might be impacting them now. In Chapter 2, I noted that Sfard and Prusak (2005) developed their framework to answer two questions. Individual coding helped me to answer the first question: "*Why do different individuals act differently in the same situation?"* (p. 14). Having the freedom to code individually led me to uncover patterns that I would otherwise have missed. For example, if I had attempted to impose the same code book across all of the data, I would not have seen that strategies for self-protection show up differently for each of them, as I describe more fully in Chapters 4 and 5. At the same time, when I looked across the final version of each participant's codebook, I found that the thematic and synthesizing level codes were more similar than I had expected. This indicates that my participants had experiences that on the surface may seem disparate, but at a deeper level there is a connection. This helped me to answer the second question: "*And why, differences notwithstanding, do different individuals' actions often reveal a distinct family resemblance?*" (p. 14). I answer these questions more fully in Chapter 5.

Coding Example

Each participant's codebook included the thematic code *messages*, which captured any messages my participants received from others or told themselves related to their mathematical identity development, as well as references to the ways that they interpreted messages. For example, one participant described being told both at home and in school that she is smart and can learn but needs extra help to learn. For this particular participant, I put the in-vivo indexing code "*I can do it if I get extra help,*" under the synthesizing code *Self-Assessment*; I put the indexing code *parents and grandmother told her she is smart and can learn but it takes her longer* under the synthesizing code *Out-of-School*; I put the in-vivo index code "*I'm not bad at math, the more I practice the more I will get it,*" under the synthesizing code *In-School* because she told me that is what she heard from her teachers and support staff; I put the indexing code *placed in a class for students who needed extra help* under *institutional messages*; and I put *attributed her successful learning to her grandmother* under *interpreting messages*. I show these as well as some additional examples in Table 3.1.

Table 3-1
Examples of Some Indexing Codes Grouped Under Thematic Code Messages and Associated Synthesizing Codes

Messages	
Examples of Indexing Codes from Several Participants	**Associated Synthesizing Codes**
"I can do it if I get extra help" "I caught on pretty fast." (Elementary School) Started to struggle (5th grade) Takes longer and is harder to learn math (7th) "I am not good at it" "I hate math, but I know I can do it." [An example of double coding with feelings toward math] "I got a C+ cause the teacher was bad."	Self-Assessment

Parents and grandmother told her she is smart and can learn but it takes her longer. "You're very smart." (Elementary School) Good at math (6th) Mother assigned extra math as a chore. You can do it from educational TV.	Out-of-School (If all out-of-school messages were from parents, this code was replaced with "Parents.")
"I'm not bad at math, the more I practice the more I will get it." Peers said math is easy. Good student (teachers) Peers underestimated him.	In-School (Teachers, Peers, Admin.)
Placed in a class for students who needed extra help Low grades Mainstream math is too hard. Felt judged by 5th grade teacher. Low	Institutional Messages
Attributed her successful learning to her grandmother Smart means I do not need to work hard or do homework. Accepted messages from failing grades, not from parents' reassurance. "I don't need you to speak slower … Just talk to me like I am a person." Wait a year for college means not good at math.	Interpreting Messages

I have 10 participants which is a very large number for a portraiture study due to the intensive amount of time required not only to develop separate code lists but to write in-depth portraits as well. So, I decided to choose several participants to be subjects of in-depth portraits. These focal participants are not a representative sample of the population of developmental mathematics students; however, I did intentionally choose them to show a range of past experiences and the ways that these experiences impacted them as math learners.

In Chapter 4, I explain the process of how I used my conceptual framework to determine which participants to focus on as well as the number of focal participants I would need to achieve my goals. I include each of my ten participants in the discussion of my findings in Chapter 5.

Authenticity

As a portraitist, I want to capture the experiences and perspectives of my participants in context to create an authentic portrait. But to do so, I must recognize that I am influenced by what Lawrence Lightfoot and Davis (1997) refer to as "voice as autobiography." Although my own prior learning and life experiences may serve as my source of motivation to do this study, I must acknowledge that they can serve as a source of bias. Because many of my participants were my own students prior to the study, I must also maintain awareness that I may be influenced by my past relationship with them and draw only from the data that I have. I must make every effort to acknowledge the privileges I have had and understand that there may be additional privileges that continue to be invisible to me and may obscure my view as I critique my own work.

Voice as Autobiography

In this section, I discuss how my history as a math learner and as a math instructor serves to motivate me, helps me relate to my participants and influences how I have designed and analyzed this study. I identify as a White woman from a mixed socio-economic status background. I am the granddaughter of Polish immigrants. Although I have experienced some obstacles along the way, overall, I have had a life of opportunity and privilege. In recent years, I have come to see that many of these privileges were invisible to me for much of my life and that there may be more that I have not yet figured out. Some of the obstacles I have faced have

ultimately served as resources for me, but I am mindful that others may experience obstacles and privileges differently than I do. I discuss a few of the most relevant aspects of my history here.

I identify strongly as someone who loves math. My earliest memory of doing math is of a moment of pure joy in second grade when I realized that I would get to do math every day in school! I remember deciding that I would become a second grade math teacher when I grew up so that I could always do math and share the joy. I have told that story many times and have often been asked for more details. But that is the extent of this childhood memory. I find that many of my own memories center on how I felt, not on what happened. I found that to be the case with many of my participants whose childhood memories are also limited, and often associated with emotion.

My mother liked math too and I often heard her enthusiastically tell people that "Carolyn likes math." She always talked about college as a given, which gave me the privilege of assuming that I *could* become a math teacher. I did go to college for a degree in mathematics, but then I went to an engineering school where I took undergraduate engineering courses before getting an M.S. in mechanical engineering. Looking back at the path I took I realize I had somehow been shielded from the myth that girls are not good at math while growing up. Or at least I never *received* that message. Instead, my early positive experiences as a math learner instilled in me a love for the subject and a strong sense that I was capable of excelling in it.

I did have some, perhaps many, gendered experiences in engineering school and while working as an engineer. But I had many years of positive messages to fall back on. The sexism I encountered in the field could not break me down because I brought a well-developed positive sense of self-efficacy within a strong identity as one who loves math and is good at math. With a different background, even with equal interest and ability, I might have dropped out.

In addition to my earlier positive experiences as a math learner and doer, early in my engineering career I had a mentor who treated me the same way that he treated the male engineers. He taught me the most important and transferrable skill that I learned as an engineer: he emphasized the importance of always looking below the surface to find the root cause of a problem. He stressed that taking the time to thoroughly understand how a mechanical/electrical system works before proposing a change is critically important for optimal efficiency and to avoid unexpected consequences. This lesson greatly influenced my decision to focus on an important but neglected part of the system of developmental mathematics. Much of the research on developmental mathematics seems superficial to me. I am concerned that new policies are being implemented in the absence of a research agenda that includes teaching and learning. The orientation I developed as an engineer of always trying to first understand the system led me to focus on understanding the identities and other aspects of mathematics-related affect that developmental mathematics students bring with them to college.

After working as an engineer for 13 years, I decided to take time off to focus on raising my children. As my children became more independent and almost 25 years after my last mathematics class, I decided to return to my passion and went back to school for a master's degree so that I could teach community college math. I started with a review of undergraduate courses which could be considered developmental from a graduate point of view. The first few classes were particularly tough; I had so much to relearn and so many knowledge gaps to fill. But my love of the subject and strong identity as a learner of math helped me through it. Like me, a few of my study participants as well as other former students had been out of school and did not take math classes for up to a decade before enrolling in developmental mathematics. They had many gaps to fill as well. Even students who enrolled directly after high school have learning

gaps. Because of this, I tried to use my own experience as a resource without assuming that I understand all the ways that they have experienced learning gaps.

Soon after graduation, I started teaching mathematics part-time at two different colleges. Most of the classes I taught were developmental, although two were credit bearing. Teaching developmental mathematics has been one of the greatest privileges of my life. I met students from many walks of life, often very different from my own, and learned so much from them as their instructor. But as their instructor, it would have been inappropriate for me to probe them about their past. As a researcher, I can now ask deeper questions and, hopefully, use the answers I find to improve the instruction these students encounter in developmental mathematics.

When I first began this study, I explored my past in detail for potential areas of bias, especially due to unrecognized privilege. But it was often the case that while writing post-interview notes, a related memory came to mind showing me that I had one more thing to be aware of. Perhaps of most significance was after my interview with Sarah when I realized that, rather than raising new questions, I finally had an answer to a question I had been wondering about for some time: "Why would a student rather fail a class than ask for help"? It became apparent to me that many students have experienced feeling negatively judged in ways that I had not. I now understand that the fact that I did not already have an answer to that question was a result of unrealized privilege. My privilege includes things that I have *not* experienced. I have not experienced having someone talk down to me for not knowing math. I have never been treated as though I was "stupid" in math. I have never been assumed to be inferior due to race or immigration status. I already had a well-developed identity to rely on before entering the engineering field. In other words, I could ask for help when I needed it without perceiving a threat to my competence.

My Own Students as Participants

Because some of the participants were my former students, I needed to be aware that my past views and perceptions of them, positive or negative, had the potential to unduly shape my interpretations of their experiences, and do my best to set those aside. During the interviews, I learned many things about participants who were my former students and their past experiences that I did not previously know, and in all cases I saw a very different side of them—a side that I could not see from my vantage point as their instructor. So, I worked to see them primarily as my participants, rather than as my former students. I found that coding close to the data helped with this prospective because it kept me focused on what the participants said or did in that small section of data.

I believe that the benefits of including my prior students outweighs the drawbacks. I capitalized on the fact that they knew me and felt reasonably comfortable with me from the outset. Most of my former students told me very personal things that I do not believe they would have told a stranger. I made it a point to approach the data with an open mind by carefully setting aside my prior memories of them. This includes my in-class observations of them as well as things they told me outside of class but did not bring up during the interview.

Chapter 4
Portraits

In this chapter, I present my findings, primarily in the form of portraits of my focal participants, followed by brief summaries of some key features of my non-focal participants. Before presenting these portraits and summaries, I present the demographics of all 10 of my participants and describe how I chose four to be focal participants for whom I wrote detailed portraits.

The demographics of my participants is shown in Table 4.1. I used my participants own words from the background questionnaire and interview transcripts to fill in this table. In terms of race and gender, two participants identified as either Black or African-American males; two identified as African-American females; one identified as White non-binary; one identified as a White male; three identified as White females and one identified as a Caucasian female. My participants ranged in age from 18 to 30. Nine participants speak English as a first language; for one it is not a first language. Three participants received financial aid at the time of enrollment in developmental mathematics, and one additional participant was in the process of applying. One participant was reimbursed by her employer, and one received tuition payment as a military benefit; one had received financial aid in the past but was denied more recently because of the break between her first and current college enrollment. Only two participants answered "Yes" to the question "Are you first in your family to go to college?" This shows that "yes/no" responses to this question can be misleading. When participants were asked if they have a disability; four

participants answered "No," five indicated that they do have one and one did not answer this question.

Table 4-1
Demographic Data of Participants

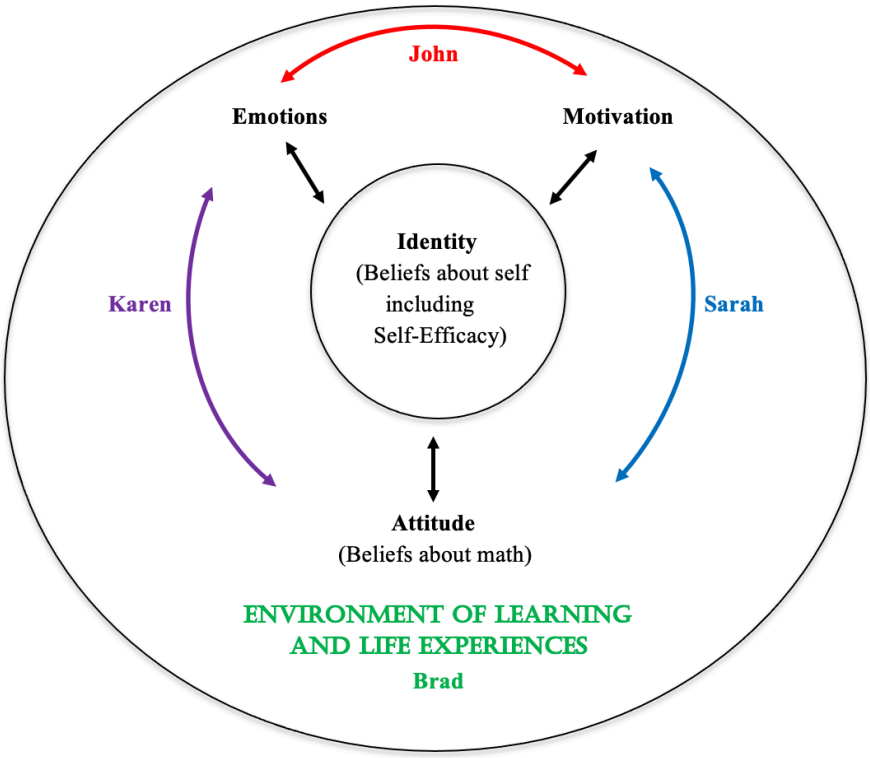
Pseudonym	Age	Gender	Race/ Ethnicity	English as 1st Language	Financial Aid	1st in Family	Disability
Brad	21	Male	Black	Yes	Yes	No	No
Karen	28	Non-binary	White	Yes	Not at the time	No	Yes Bipolar II
Sarah	18	Female	African American	No	No But applying	No	Yes Deaf in 1 ear
John	20	Male	White	Yes	No	No	Low Vision
Victoria	20	Female	Caucasian	Yes	Yes	No	No
Kelly	19	Female	African American	Yes	No	No	No
Mary	19	Female	White	Yes	Yes	Yes	[Did not Answer]
Ashley	19	Female	White	Yes	No	Yes, but Sister	Yes Learning Disability
Pearl	30	Female	White	Yes	Employer Pays	No	Yes Learning Disability
Elijah	25	Male	African American	Yes	Military Pays	No	No

Note: These entries reflect participants' own words. The first four participants to be presented on this list were focal participants.

How I Choose Focal Participants

As I noted in Chapter 3, after I collected and coded data from each of my 10 participants, I had the task of choosing a smaller number of participants for in-depth portraits. To do so, I used my conceptual framework to help me identify participants whose portraits would enable me to most fully answer my research question. As I reviewed my coded data, I noticed that each participant highlighted all aspects of the framework to some extent. As I further considered each participant as an individual, I noticed the ways that each of four participants brought out the importance of different aspects of the framework. Figure 4.1 shows how I mapped each of these four participants onto my framework. I first noticed that **John**, in particular, illuminated the ways that emotions can interact with identity and interact with motivations. Next, I noticed that **Karen's** narratives highlighted the ways that one's beliefs about the importance of math interact with self-beliefs and together interact with emotions. **Sarah** revealed that motivation to learn math can interact with and combine with beliefs about the importance of mathematics and together interact with self-beliefs. But perhaps the most powerful narratives are those that **Brad** generously shared about the ways that the environment impacts learning and influences all aspects of mathematics-related affect.

Figure 4.1: *Mapping My Focal Participants onto My Conceptual Framework.*



Although I used a process to deliberately choose focal participants, they do not represent the population of developmental mathematics students. Rather, the portraits of these participants help us to understand the importance of the ways that learning is influenced by one's identity, attitude, emotions, motivation and environment, as well as the ways that these elements combine and impact learning.

Developing Portraits

While developing portraits, I considered each participant as an individual to help me understand them as math learners in the context of their past learning and life experiences. I sought to understand the nature of how their past learning experiences shaped their identities, how their identities developed over time along with other mathematics-related affect and how these things impacted their actions and learning. I examined the sources and nature of messages

my participants have received about themselves as math learners, the consistency of these messages among sources and over time as well as the participant's acceptance and rejection of messages.

As I considered the data as a whole for each participant, I noticed and wrote about themes for each individual. It was often the case that seeing something that was foregrounded by one showed me the way that another participant was impacted by something I might have otherwise overlooked. For example, Karen described intentionally trying to develop apathy as a strategy for coping with the pain of failure which I refer to as a "strategy for self-protection." This helped me to understand that when Brad tried to make people laugh in school, it was a strategy for protecting himself from social isolation.

The themes, reflected in the subheadings, are not the same for all participants, although there are some similarities. The themes reflect what was most salient to the particular participant. For example, all participants received identity messages and developed identities as math learners. However, the ways in which identity messages were received and assessed varied among participants. The timing of changes in messages and consistency of messages also varied among participants. Both John and Sarah were reluctant to seek help with math when needed, which was a strategy each developed for protecting themselves from negative judgment. However, the sources of negative messages they received that led to this strategy were different and the ways that they overcame this reluctance was different as reflected in the narratives and themes.

John: "What if they judge me? What if they think I'm stupid?"

John brought a range of emotions from his past learning and life experiences to his college developmental mathematics courses. These included anxiety, fear of judgment, and

shame on one end; and feelings of gratitude and deep appreciation for those who supported him throughout his K-12 years on the other. During our interview, I found him to be a warm and passionate person whose career goal is to help others as a social worker. John has a significant vision impairment and has been approved for accommodations since 3rd grade. As a college student, his instructors would be informed that he has been approved for accommodations. College demographics would reflect an additional student with a disability. But his instructors, administrators and researchers would not know the way that his past learning and life experiences, especially those related to his low vision, have shaped his identity and resulted in strong fear of judgment that makes it difficult for him to ask for help. They would not know that throughout his education he often had to advocate for himself to receive the accommodations that he was legally entitled to.

John identifies as a White male who was 20 years of age at the time of the interview. English is his first language. He comes from a highly-educated family and does not receive financial aid. John also has a severe visual impairment, which he refers to as "low vision," resulting from albinism. He cannot read standard print even with corrective eyewear. Even when print is enlarged to the point that he can read it, reading is still strenuous, and he becomes fatigued quickly. But he is also highly motivated and pushes himself to persevere.

Pre-Diagnosis Identity and Emotional Development

John's identity as a math learner began to develop years before the extent of his vision impairment was known and it was closely connected to his emotions. He felt stressed watching his peers doing math because, from his perspective, they were able to do math quickly and easily while he was not. Although John's earliest memories include wearing corrective glasses, the extent of his vision impairment was not known until the end of 3rd grade. He did not realize when

he was young that he could not see what his peers could see: “Because I was born with it, I just thought everyone saw like I did.” He could not understand how his peers knew when to add and when to subtract which he found confusing: “I remember … I would always get addition and subtraction mixed up. It was because I'd always mis-see the symbol and I would think the plus was a minus and the minus was a plus.” Time-limited tests in particular were a significant source of anxiety as he strained to read.

At the same time that he was struggling to learn math, he began to hear in school that math is essential in life. This added to his anxiety: “I felt really pressured and stupid because I just couldn’t understand.” John remembers feeling despair:

> I don't know what I'm gonna do in my future if I don't know how to like, all my friends know how to add; I don't know how to add. All my friends know how to subtract; I don't know how to subtract.

During this pre-diagnosis phase, John received messages from home about the importance of college. His parents and grandparents are college educated and his family emphasized the importance as well as expectation of a college education: “I don’t remember a time when my parents had not talked about college … my grandparents were doctors and they were very big on college.” His school emphasized the importance of education, and of math in particular, as well. All of this intensified his feelings of anxiety.

John has a twin sister who he described as a constant source of support. Even at a very early age, she assured him: “‘You're gonna get into college and you're gonna get into a good school. You're gonna do well. It's gonna be okay.’” He received assurances from his mother as well. He was thankful for their encouragement and the confidence that they expressed in his ability to succeed. But their words were often eclipsed by his self-assessment that he did not

understand math and was behind his peers and resulting feelings of anxiety. This combination led him to question his ability to succeed: “I don’t know how I’m ever gonna know this.”

At some point during 3rd grade, his mother began to notice that after school he was often exhausted, his eyes were bloodshot, and he had frequent headaches. She mistakenly thought it had something to do with his brain. So, he was evaluated and “it was nothing.” His mother’s friend, who was a school social worker, suggested that the symptoms could be related to his vision. From there, he was diagnosed by two specialists as having albinism, which affected his vision. At the time of his diagnosis, school math included multiplication, division and long division but he was still struggling with addition and subtraction. During the spring of 3rd grade, he was formally approved to receive accommodations including enlarged print, having tests read to him, extended time on tests and use of technology. But he still had to re-learn math that had been taught earlier. He found the extended test time to be the most helpful because even with the other accommodations he still felt anxious about being able to complete his work on time which led to feeling defeated.

Post-Diagnosis: Concerns About His Future and Fear of Being Judged by Teachers

John’s diagnosis helped him to understand why learning had been more difficult for him than for his peers but did little to lessen his feelings of self-doubt as he continued struggling to learn. He recalled feeling that “5th grade was rough.” He said that when he asked his teacher for help, he felt judged: “She was like, ‘You should know this.’ I was like ‘But I don’t.’” He continued to have difficulty asking for help because he felt that whoever he asked would judge him:

> I think for a while, I was just really scared to ask people for help, the teachers especially, because I would just get in their way or they would think I should know this, and they would judge me.

His anxiety over what his teachers would think of him if he asked for help continued into his high school years. He further explained: “It was just my own fear of, ‘What if I get in their way? What if they judge me? What if they think I’m stupid?’”

Hearing about the importance of math combined with his doubts about his ability to learn math, he became concerned about his future. In particular, while he was struggling to understand money, he felt terrified as the teacher warned the class about the consequences of not learning about money. He remembered what she told the class and how it impacted him:

> ‘If you don't learn about money, if you don't learn how to count money, then you will not make it into life. You won't make it into college and you'll probably become homeless.’ That's almost the exact words and that shook me to the core. “I was like, ‘Homeless?’”

John’s mother was sympathetic to his fears and hired tutors to work with him. When he started middle school, his tutor encouraged him to use his accommodations. He realized that he was hesitant to ask teachers for help due to a fear of judgment that started in 5th grade. His fear of judgment and feeling that he might be perceived as a nuisance became part of his identity and continued through middle school and high school.

Fear of Being Judged by Peers

John’s concerns about how others perceived him took a turn in terms of who he feared would judge him during his high school years. In earlier grades, he noticed that his friends and twin sister knew more math than he did. During 5th grade and throughout middle school, he feared that adults whose job was to help him learn math might judge him. But once he got to high school, he also became concerned about what his peers and his twin sister would think of him. From John’s perspective, his circle of friends, including his sister, were very intelligent. For example, his sister took a lot of AP and other advanced classes in high school while he took remedial courses.

High School Coursework as a Source of Shame

John was constantly aware that the courses he took were not at the same level as the courses his friends and sister took. He explained that he enrolled in a mainstream algebra course during the first semester of 9th grade, “like everyone does when they’re a freshman in high school.” The course was too difficult for him and he did not pass. So, he was placed in a resource room class for the second semester. He found this class to be slower paced and very easy; he passed the class with minimal effort:

> I don't know if it was the instructor, I don't know if it was the environment, but it was just a blow off class. There was no effort put into it. We had no homework either. We had zero math homework. No one said to go practice.

John also felt that the teacher did not care about the students, had low expectations and did not assign homework:

> She was retiring the next year, so it felt like she didn't care about her job at all. We barely did any work in that class at all, but it was just super easy and almost like a blow off class.

A counselor, that he referred to as his caseload teacher, agreed that his next placement could be a cohort class, which he described as in between a resource room class and a mainstream class. For each of the next two years, he was enrolled in a cohort class where he took Algebra I and Geometry with a teacher that he thought highly of: “I think she was a good mix of stern, but also understanding.” But he struggled and barely passed Geometry, so his caseload teacher recommended another resource room class. John was concerned about falling further behind his peers and pleaded with her to continue in a cohort level class:

> My caseload teacher was like, ‘You need to take another resource room math class,’ and I didn't wanna do that because I knew that would be even more behind if I take this class. I literally begged her, I'm like, "Just please put me in this cohort class. I will do better. I will work hard." And that's what I did. I worked hard.

John had always compared himself as a math learner to his friends and twin sister. At one point, his sister looked at his class schedule and asked: "'Is that remedial stuff where all the bad kids go?'" He was embarrassed about his high school class schedule and was concerned that "everyone would think I was dumb." As a result, he was hesitant to let his friends know which classes he was taking. He was often in class with younger students and always felt that he was a grade behind which led to strong negative emotions including shame: "I was very ashamed of the class route that I took in high school." As he finished high school, John feared judgment not only from authority figures but from his peers as well.

Anxiety Triggers

John needed to take a second math class during his final semester of high school because of the class that he failed during his freshman year. So, he enrolled in a personal finance class. He liked that he was in a math class with students his own age for the first time in years. He also felt that the math was personally beneficial. But he struggled in a different way in that class than he had in other math classes. He described the math itself as less difficult, but at the same time it was a greater struggle for him: "It wasn't even that it was difficult math. It was just life math. It was so difficult. Words can't describe how much I struggled in that class." So, although the math content was not more difficult, he experienced a different and much higher level of anxiety. The anxiety he felt in his other high school math classes was related to learning. This class reminded him of his 5th grade teacher's warning that not learning about money would lead to not doing well in life, including the possibility of becoming homeless:

> I don't know. It was a different type of anxiety with my other math classes 'cause at least with my other math classes there was that drive like, "I want to get this right." But then I walked into personal finance and I don't even know. It almost took me back to like, the 5th grade, where I had no clue what I was doing with regards to math. Yeah, I just wanted to hide under the covers and just pretend it's not even there. Yeah, that was rough.

Gratitude

John frequently expressed feelings of deep gratitude for those who have supported him over the years, even as early as 3rd grade. After his low vision was diagnosed, he was allowed to receive extra test time as part of his accommodations. But to use this accommodation, he had to miss recess to take tests. He also missed recess to work on the math that he missed prior to his diagnosis. Rather than complain, he expressed gratitude for the opportunity:

> They let me have as much time as I needed to take. I would sometimes go in during recess and take it. I had a great third grade teacher and she would practice stuff with me during recess and everything.

Although he had to regularly advocate for his accommodations, some teachers were exceptionally supportive. He explained that in addition to having low vision, he has a family history of mental illness. The teacher he had for both 10th and 11th grade math believed in him and was supportive at a time when he lost motivation for school and wanted to drop out:

> My family has a history of dealing with mental illness and I think that's why my math teacher was so great during that time because she knew I was capable and throughout that time I just had no motivation to do anything. My grades were slipping, especially my sophomore year. It was so rough because I lost motivation for almost everything. Even for theater and choir, I was just losing motivation. I was like, "I don't wanna be in school anymore. I just want to take the easy way out and go to some sort of remedial school."

John went through a particularly difficult time at the beginning of high school when his parents got divorced. He explained: **"**I've dealt with depression. I've always had depression, but when my mom and dad got divorced, it was so rough." Choir and theater provided him with an essential support system which he described as an escape during tough times:

> Choir and theater were my excuses to go to school and wake up in the morning 'cause I had such a great system there. There was a few of my closest friends; I knew these people since preschool, were in theater with me and they were the only people that knew about my struggles throughout high school and with class and everything. They were there for me and they helped me.

John also had immense appreciation for the choir teacher he had all four years of high school. When he was going through a period of intense doubt during his sophomore year, she assured him: "'You are so capable. You're gonna make it. It's okay. I know you're down on yourself and you're gonna be someone and you're gonna be great.' That really, it was like, wow." John felt that the support he received from his choir teacher along with his choir and theater friends helped make his last two years of high school really great and helped him to experience a lot of growth.

John also felt that he had really great parents. But the person for whom he expressed the most gratitude, was his twin sister. He felt close to her and felt her love and emotional support throughout his life:

> Yeah, I have a little sister and a twin sister. I love my little sister, but my twin sister was my support. She would get mad at me sometimes for being just who I was; kinda goofy around her, but she was like, if I ever have anything at all, my twin sister was right there. Through a period of my life, I was like ... Yeah, beginning of high school, my parents went through a really messy divorce and my sister was there and she was like, I can't describe. She was like, so close to me. Yeah.

Strategies for Self-Protection

John protected himself from fear of judgment in two opposing ways. On one hand, he often avoided asking for the help he needed and avoided asking for the accommodations he was legally entitled to. On the other, his feelings of gratitude for those who supported him provided motivation to push through tough times.

Although John had accommodations in place for his disability starting in the spring of 3rd grade, they were not provided to him automatically as they should have been. He had to advocate for himself. He remembered that starting in middle school, he received messages from his parents that he needed to advocate for himself. However, his hesitation about asking for help was rooted in his 5th grade experience: "I was afraid I was gonna piss people off if I asked for help. I

don’t know. I think this goes back to the 5th grade.” These feelings continued throughout high school. Although he described his accommodations as being provided more regularly than in previous years, he still needed to ask for some accommodations including extra test time, which he was hesitant to do.

His mother reprimanded him and told him he would not graduate if he did not ask for help. While this did make an impression on him and provided some motivation for him to push himself to ask for his accommodations, a turning point came when a vision counselor told him there would be consequences for those who did not comply with his requests. This helped him to feel empowered:

> I think what really set the mark, I think one of my vision counselors was like, "If they don't do your accommodations, they could get in real big trouble." I'm like, "Well, okay." (laughs) I literally asked for extra test time and they're like, "Yeah, yeah." And I'm like, ‘Okay, this is easy.’

John also protected himself by balancing his fear of judgment with feelings of deep appreciation for those who supported him in his learning as well as those who helped him through difficult times. Whether this was an intentional strategy or not, John’s deep feelings of gratitude outweighed his fears and provided him with the motivation he needed to face challenges. Throughout the interview, as John described the challenges he faced, he also acknowledged the support he received. With gratitude and pride, John explained that after many periods of self-doubt and wanting to give-up, he graduated from high school and was hopeful for his future: “Then when I graduated, I didn't think I was gonna graduate and I did, and it was just so ... I felt like I was actually going somewhere with my life. It was really great.”

Karen: “I was so anxious that … I decided not to care about it.”

Karen came across to me as is a particularly self-reflective person whose narratives included powerful insights into the many ways that someone’s actions could be misunderstood

and misinterpreted. Throughout our interview, Karen provided alternative explanations for what others viewed as underachievement and laziness. Karen also illustrated how an intentional turn to apathy helped to ease the pain of being misunderstood.

Karen identifies as nonbinary and uses the pronouns *they* and *them*. During our interview, their mood was calm and relaxed but, at the same time, I sensed an edge of anticipation. Karen had chosen to print out and bring a hard copy of their completed background questionnaire to the interview. As I read through the questionnaire before beginning the interview, I realized the need for hand delivery and an extra layer of privacy. When I reached the final page containing answers to questions concerning disability and its impact on learning, I felt their gaze intensify as a diagnosis of mental illness was revealed. During the interview, I learned that Karen's learning and life experiences were substantially and often negatively impacted by a mental illness that was unrecognized and undiagnosed for most of their life.

Karen identifies as a White person, speaks English as a first language, and was 28 years of age at the time of the interview. They had attended public schools throughout K-12 and had been raised by college educated parents. Karen was never classified as having a disability and had never received accommodations. However, the symptoms of a recently diagnosed mental health disorder have interfered with their life and learning throughout K-12 as well as their initial postsecondary schooling. Karen felt pressured to attend college immediately after completing high school. However, they did not feel up to the task, dropped out, and went to beauty school instead. They worked as a hairdresser for ten years, on their feet for hours at a time, which required a level of energy that they felt they could not sustain. Motivated by the need for a career change, they enrolled part-time at Midwestern Community College (MWCC) and continued to

work part-time as a hairdresser. Although they were only working part-time, due to the need to retake some classes after the long break, Karen was temporarily ineligible for financial aid.

Karen received verbal messages about themselves from teachers and parents as well as messages from test scores and grades throughout their K-12 years. Initially, messages from home and school were consistent and positive, and they accepted what they heard without much thought. But as messages began to change, Karen had to make assessments and decide which messages to accept or reject. They enjoyed math and their attitude toward it remained positive even as their self-efficacy, and overall identity as a math learner, declined.

Key Transition Points of Identity Development

Karen's narrative reveals strands that shaped their past learning and life experiences: their identity as a math learner, beliefs about math, messages they heard from others and mental health. It also reveals that these elements transitioned simultaneously at key transition points. Because of this, I present their identity development around several phases of learning. The first phase begins with their earliest memories of math, starting in 3rd grade and lasting through 6th grade. During this period, each strand was stable and positive with only one memorable, but brief, exception. A key transition point occurred during 7th grade, marking the beginning of the second phase which lasted through 8th grade. Karen began to experience difficulty learning math which impacted and was impacted by messages they received about themselves as a math learner. During that same period, symptoms of an unrecognized mental health disorder began to appear along with a high level of anxiety, both of which intensified learning difficulties. The third phase of learning includes their high school years and initial college enrollment. They became deeply depressed, which lasted throughout most of this phase. The depression caused Karen to experience low energy, which was interpreted by others as laziness. During this time,

Karen felt hurt by these accusations of laziness. Finally, receiving a diagnosis marked a positive transition point at the age of 27, because it meant getting needed medical help. Within two years of receiving medical treatment, Karen completed the associate degree that they had first attempted more than 10 years before.

The Early Years (Grades 3 – 6)

During the earliest phase of mathematical identity development, the strands that shaped Karen as a math learner and influenced their learning were in alignment and almost inseparable. Karen enjoyed learning math during this time. They felt math was fun and easy and that they learned arithmetic quickly and effortlessly. This self-assessment was reinforced by messages from parents and teachers that they were smart. Together, these strands led to their childhood conclusion: they were smart because they learn easily. Their experiences with math were aligned with their experiences in other subjects and with school more generally. Their actions supported their assessments of these strands.

Karen's feelings toward math, their self-assessment and their beliefs about math were strongly related during this phase. Karen liked math. They recalled: "I did enjoy math. I enjoyed school a lot when I was little," and "I remember liking math class through elementary school." Their pleasant memories of doing arithmetic include multiplication and long division. Along with this enjoyment they had a sense of their own ability which they described as: "I'm very smart," "I caught on pretty fast in elementary school" and "At one point I had a pretty good understanding of long division." They believed math was easy: "I was telling myself that it was easy, the stuff that we were doing."

One exception to the ease of learning during their early years was a time of struggle in 5th grade. As an outward sign of frustration, Karen started to grind holes into their math book: "I do

remember I had my math textbook that I ended up having just a lot of holes in it 'cause I would get frustrated and I would be grinding my pencil." A teacher whom they thought highly of helped ease their feelings of frustration by providing individual instruction: "My fifth grade teacher, he was a really good teacher and he always took time to give his students individual attention when we were struggling with things." Karen offered no other memories of this early experience.

Although intense frustration became a problem during the next phase, their overwhelming experience with math through 6th grade was that it was enjoyable as well as easy: "I was still pretty into math and stuff, through 6th grade." Throughout this period, Karen received accolades from both teachers and parents for intelligence, which included being told that they were "smart" and "catch on quick"; they were even told that they were "gifted." "I was often told that I was a smart kid. I got that a lot." Karen absorbed these messages and accepted them as true: "I mean, I think at that age, I pretty much just believed what I was told. I was like, "Oh, I'm very smart. People are saying that to me, so that must be true." But praise for aptitude at a time when the work was not appropriately challenging for them led them to the conclusion that hard work was unnecessary:

> It was nice to be told those things, but I think it also made me like, I wasn't too focused on the *work* that goes into it. I just figured, "I'm smart. I don't need to try that hard. I'll just do the work without studying or doing the homework."

Karen was motivated enough to be engaged with math during class but saw no reason for doing homework. They did not make a connection between effort and learning. Looking back, Karen reflected that not being appropriately challenged during their early learning years laid a weak foundation for later learning: "I didn't have many study skills coming into middle and high

school, really. I was used to just blowing through it." Karen had not developed an appreciation for productive struggle and had not learned how to deal with challenge:

> Anything that took me more than two seconds to figure out, I was like, "Well, it's clearly impossible. It just can't be done," so I would give up very easy once I reached something that I couldn't immediately figure out.

Middle Years (Grades 7 – 8)

Karen experienced significant change within each strand starting in 7th grade and lasting through 8th grade. Once again, these strands were in alignment, but no longer in a positive manner. After years of ease in learning math, Karen struggled to adjust to changes in the type of math they were learning. They began to experience anxiety, rather than enjoyment, with math as their confidence in their ability was shaken. At the same time, the messages they received from their 7th grade teacher were opposite those of earlier teachers. In other words, where prior teachers had communicated that they were capable and intelligent in math, their 7th grade teacher questioned their abilities. They also experienced changes in messages about themselves from grades and from parents. While Karen tried to make sense of these new messages and experiences through the lens of the math identity they had formed during earlier schooling, emerging symptoms of a yet to be diagnosed mental illness intensified their feelings of anxiety.

During 7th grade, Karen found the transition to algebra confusing: "Letters in math? What is even happening?" They found the transition from concrete arithmetic to the abstraction of algebra to be quite difficult, especially after years of ease in learning:

> I mean it's just it wasn't coming nearly as easily to me as it had in the past. It was taking a lot longer for me to catch onto stuff and for me, I was so used to immediately catching on and blowing right through it.

In addition, Karen found that the classroom atmosphere felt negative for the first time. They described their 7th math grade teacher as "very irritable" and recalled that she frequently

berated students during class and did not allow students to ask questions during class. In-school messages changed from math teachers talking about Karen as “gifted” to the teacher calling them an “idiot,” which made a particularly harsh impression.

Although less dramatic, messages Karen received from their parents also changed. Their parents were caught off guard by the sudden change in Karen’s grades and didn’t understand why their very smart child was suddenly failing math. Karen described their parents as “still very encouraging” but at the same time, confused:

> But there was also a lot like, "You're so smart. Why are you suddenly ..." It was that I must not be working hard enough. There was some of that going on, too. It was like, “You're smart, so there's no reason you should be failing. Why are you failing this class all the sudden”?

Their parents’ message only helped to intensify Karen’s prior conclusion that finding math to be easy means one is smart. But if ease of learning means someone is smart, then having so much difficulty must mean they are no longer smart: “It's too hard. I'm not smart enough. I can't do it anymore.”

As the work became more challenging and the messages Karen received became more negative, they began to feel increasing anxiety during math class. To ease the pain of this anxiety, Karen tried to stop caring. This approach, when combined with struggles they experienced around the content, made math less enjoyable and engaging. They described how the teacher provided no help and moved along regardless whether students were learning:

> Especially getting no support from the teacher, it was like I didn't have anybody to help me or ask questions to about it, so I just kinda gave up. I was like, “Well, this is as good at math as I'm gonna get, I guess.”

During this time period, Karen felt particular pain and anxiety around taking math tests. They explained: “Now we've moved onto something I really don't understand, and I know a test is coming and I don't understand what we're doing. I would get very, very anxious.” For Karen,

not understanding the math was a major source of test anxiety. At this point, Karen began to disengage from math.

Although the classroom atmosphere improved in 8th grade, Karen's disengagement during 7th grade carried over. They felt discouraged, stopped paying attention in class, and stopped doing homework, stating: "I was completely disengaged as a math student at that point." Their 8th grade math teacher expressed frustration with Karen and attributed their poor performance to not doing homework, rather than intelligence. Karen described their 8th grade experience:

> I would test okay often. My tests would be okay, but I wasn't doing any of the homework or any of that kind of stuff, which was really negatively affecting my grades, which was very frustrating for the teachers 'cause they're like, "I can see that you can do this stuff. You're just not doing it." So, they found that very frustrating.

Their parents' response to failing grades on report cards was that Karen was "disappointing" them. Karen remembers that their parents told them: "We know you can do better." But messages they received from the failing grades were more powerful and Karen concluded that they could not do better: "I'm like, 'Well, I don't,' so I didn't."

High School and Initial Postsecondary Enrollment

By high school, the strands that had shaped Karen as a math learner and influenced their learning were now more significantly misaligned and causing tension and pain during the third phase. Karen reported that they had very little energy. Although the reason for this was unknown at the time, it impacted every aspect of their learning and life. They did not have the energy to stay awake during class or to do homework. Their parents expressed disappointment and attributed their failing grades to laziness. Teachers had little to say but Karen received messages from failing grades.

They knew that they were not lazy and wanted to put forth effort, but they were unable to do so. They felt discouraged and tired but did find math to be interesting and enjoyable. Karen brought up enjoying math during high school several times. They shared that they did increase their effort for a time, which led to better understanding of algebra and enjoyment of math. But in general, they were too tired to care about school and gave up easily on homework. The symptoms of their mental illness were powerful: “I did enjoy math. I liked algebra and I thought it was fun and interesting and all that, but it was also a lot of work that I didn't have the energy for.”

They wouldn’t learn until more than 10 years later that their adolescent lethargy was a symptom of deep depression. They shared their pre-diagnosis memories with me post-diagnosis; often intertwined with post-diagnosis reflections:

> I was also struggling quite a bit with undiagnosed mental illness in high school, so I was very, very depressed often, and didn't have the energy to do much of anything in any of my classes. So, I was doing poorly in math, but that wasn't just math. I was doing poorly in all of the classes.
>
> Another significant symptom was excessive daytime sleepiness.
>
> I slept through the majority of my classes. I was sleeping all day and then I'd come home and sleep some more, so I wasn't doing homework. Then I was also working a job in high school, so I had no energy for anything.

In addition, high school class sizes were large and there was little individual attention. If teachers and others at school sent messages about them as a math learner during high school, Karen did not receive them. Although, even in the absence of verbal messages, their failing grades were loud and clear. Further, they continued to receive messages from their parents that they were “disappointing” even though there was no indication of any communication between home and school. Their parents concluded that Karen was lazy: “Often it was treated as like a laziness

issue. That I was just deliberately not doing things because I was lazy and loved failing at school." Karen didn't have the energy to do much at that time, including fight the accusation.

Karen did not feel up to going to college immediately after high school but did so under pressure from their parents. They did not enroll in any math courses at that time, but the classes they did take did not go well and they "immediately flunked out." They added "It took a long time before I decided to tackle it again, so it was a lot of money and time wasted."

Reenrollment, Diagnosis, Persistence

Nearly a decade after their first attempt at college, Karen was motivated to enroll in college for the second time by the need for a career change. This time, Karen enrolled at Midwestern Community College, which was not the same college as their initial enrollment. Karen decided to take a mathematics class during their first semester and took a placement test. Their ability to guess well on multiple choice tests led to an artificially high score which resulted in over-placement. As Karen explained: "I was in way over my head. I started failing the tests very quickly in that class and dropping out." This time, failure had a devastating effect on their mental health, and a diagnosis soon followed: "I had failed out of the math class and was feeling very low. I actually ended up institutionalized for about a week or so, and that's where I got the diagnosis. Finally, I guess."

Karen was diagnosed with bipolar II disorder. In their own words, the main symptom of this disorder is severe and prolonged depression which can interfere with all aspects of a person's life. More specifically, Karen described bipolar II as "mostly depression with some small spikes of mania that aren't as severe as you get with bipolar I. It's mostly the depression which is the hard part." An important consequence of the disorder is that it "can make simple daily life tasks difficult let along trying to balance work and school."

> It makes it difficult to do things like show up to class in the first place and do the work and stuff like that. It makes it very hard to keep up when you don't have energy to do even basic things like feed yourself or get out of bed. It makes it hard to do homework on top of that, so I have to be very careful with my energy levels. I have to really manage them very carefully.

With the diagnosis came the acknowledgement that something very real was interfering with life and learning and it was not "laziness," Karen's diagnosis and treatment also gave them the tools to move on more successfully in life and learning: "It's easier to manage and I can do things like take this class without pushing myself into a crisis."

Despite what might be considered as two failed attempts at college, with the benefit of diagnosis and treatment, Karen persisted and enrolled a third time. Karen enrolled again in a MWCC math class, but this time they dropped down one level which turned out to be a more appropriate developmental math class. The lower level class went well. Karen completed the course with a very high grade, but more importantly, they felt prepared enough to re-enroll in the original math class that they had previously failed. This time, the class went well. Within two years of enrollment in an appropriate math class, Karen completed an associate degree.

Self-Protection: Efforts to Change Frustration to Apathy

Karen described their intentional effort to stop caring during 7th grade as a way of lessening the pain of being unsuccessful at something that mattered to them: "I really stopped caring. Stopped doing the homework, was playing games on my calculator instead of engaging with the material." Feeling there was no other alternative, Karen willfully turned to apathy to make the pain of anxiety more manageable:

> I was so anxious that I just shut down about it. I was like, "Well, I just don't care at all. I'm not stressed about this because it doesn't matter to me," which is one way to deal with it, I guess … I didn't really know what to do about it and felt like there wasn't really anything I could do about it, so I decided not to care about it. I was somewhat successful in that regard, yes.

Karen was somewhat successful with developing apathy and using it to replace anxiety, but not completely. Their anxiety came back both during tests and when getting test grades back. Karen felt that not knowing where to even begin to solve problems on a test confirmed that they did not understand. They also felt stressed when grades came back and confirmed that they were failing.

Misalignment Between Self-assessment and Messages from Others

Karen's memories of their past learning and life experiences present an interesting mix of emotions, self-efficacy as part of overall identity, self-perceptions of achievement, and attitude toward math. Some of these aspects of mathematics-related affect changed with the changing messages Karen received from their teachers, parents, and grades. But the messages Karen received from different sources were not aligned and did not align with Karen's own self-assessment. The only stable aspect of their prior learning was an enjoyment of mathematics. Their positive attitude toward the subject of mathematics persisted even as they tried to stop caring about doing well in math. However, their emotions changed from joy to anxiety as math became difficult and negative messages were accompanied by what Karen felt was a lack of support for their learning.

Karen's identity as a math learner involved more than an ability to learn math well. It included an internal struggle around feeling perceived as a lazy math learner. Karen felt hurt by these accusations of laziness because they always knew that they lacked the energy to do what they wanted to do and were *unable* to put forth greater effort, as opposed to the accusations of being *unwilling* to put forth the effort. After receiving medical treatment and learning how to manage their energy levels, Karen succeeded in both learning and degree completion. What others perceived as laziness began with Karen's intentional strategy to ease the pain of anxiety

by replacing it with apathy. This learned apathy was overshadowed during their high school years by the low energy and excessive daytime sleepiness which Karen can now explain as symptoms of an untreated mental illness: “I needed medical help.” I sensed a tone of exoneration in Karen’s self-reflection after receiving their diagnosis:

> I think it could've been *very* different if I had gotten a little bit of help 'cause often it was treated as like a laziness issue. That I was just deliberately not doing things because I was lazy and loved failing at school. I was definitely blamed a lot for being sick, and I feel like if things hadn't been that way, it could've been very very different. Now, getting some treatment for my illness, it's easier to manage and I can do things like take this class without pushing myself into a crisis.

Emotions also played an important role in Karen’s learning and identity development. The anxiety that accompanied Karen’s depression in both middle school and high school led to frustration and feeling incapable of learning math which in turn led to giving up easily. Anxiety impacted their thinking; they found that they were unable to think clearly and made silly errors on tests when feeling anxious.

Sarah: “Just talk to me like I’m a person.”

Sarah is highly motivated to succeed academically in order to succeed professionally. She identifies as an African American woman and as an immigrant. Her family immigrated to the U.S. from West Africa when she was 5 years old. English is her third language after French and her native language, which she did not name. She feels that she was fluent in English by 7th grade. At the time of the interview, she was 18 years old. I would not have known that English is not her first language if she had not told me. Nor would I have known that she is deaf in one ear. After she told me these things, I noticed that she sat slightly to the side so that her hearing ear was toward me. I also noticed a trace accent. But she spoke English as fluently as a native speaker.

However, speaking English so fluently now does not mean that her past learning and identity development were not impacted by the reactions of her teachers, tutors and high school counselors. Sarah described the unfounded low expectations and condescending attitudes she experienced from earlier educators, which she attributes to the way that these individuals perceived her as an immigrant. She remembers that when she asked for help with mathematics or other academics throughout K-12, teachers and tutors would speak slowly and loudly; she felt that she was being judged as having inferior intelligence. The negative experience she had with asking for help with math in the past has had a lasting impact on her level of comfort in seeking help.

Sarah attended all public schools throughout her K-12 education and enrolled in college during the fall semester that followed her high school graduation. She was raised by college educated parents. She did not receive financial aid at the time of the interview but planned to apply for aid. Sarah identified as one who is not strong in math but her beliefs about the importance of education motivate her to work hard at being successful in learning math so that she can have a successful career.

Identity Development

Sarah's beliefs about the subject of mathematics developed alongside of her beliefs about the importance of education and her beliefs about herself as a math learner. Early on, she received the message from her family that learning math was essential to progress through school and into a career. Although her parents were more directly involved in her academic life during elementary school, they always had a very strong influence on her life and identity. The strength and influence of in-school messages became stronger as she progressed through middle school and high school.

Early Identity Development: Beliefs About Self and Beliefs About Math K-7

Sarah's early memories of learning math were almost exclusively of learning at home rather than at school: "It's always me at home, on the dining room table with a family member." She learned the basics of math as well as other subjects from her parents during elementary school. She remembered her parents as being very "hands on" during her elementary school years and that they taught her math the way that they knew math. But what they worked on was not homework in the traditional sense of working on school assignments. Rather, each year from 1st through 5th grade, her mother bought supplemental math books such as *What Your Fifth Grader Needs to Know*. This supplemental learning continued during the summers as well.

Sarah learned to view math as a chore, rather than as something that could be enjoyed because it was presented to her that way. Her parents assigned pages in the workbooks that had to be done before she could watch TV. Sometimes math actually was on her chore list: "wash the dishes, sweep and then you need to do 10 pages." As a result, Sarah did not like math. When she told her parents that she did not like math, she was told that she would be held back in school if she did not learn it: "It was always like, 'You need to learn it,' and it wasn't necessarily like, 'You need to enjoy it.' It's just, 'This is something that you have to know.'" So, she came to see learning math as a necessary part of moving forward in school: "I never really learned with the intention of enjoying it, but I learned with the intention of, I don't wanna be 16 in eighth grade."

Sarah's memories of learning mathematics in school at this time are less vivid. She explained: "I barely remember elementary school, but it was ... What I do remember is like, it really wasn't anything, you know?" She described classroom learning as passive as opposed to the hands-on approach of her parents. The disconnect between the way she was taught at home and the way that she was taught in school led to confusion: "When I think about being in a

classroom it was always like, 'I don't know what to do.' I was just kind of there." She also remembers playing math-related games in school:

> It was like, math class was fun to a degree. You know, you could play games. We had relay races where it's like a whole bunch of problems and the first person goes up. You have to write it down, you have people cheering you on like, "You can do this! You got this! You know the answer!" And the next person would take the marker and the first team to win gets candy. Math, school was fun in elementary school.

Unfortunately, playing games did not lead to feeling that she actually understood the math involved.

Sarah's identity as a math learner was strongly tied to her career goals. In fact, when I specifically asked the about messages she received, about who she is as a *learner of math*, she interpreted my question in terms of what others told her she was capable of doing as a career: in school, she remembered hearing, "You can be anything you want to be." at home she was told "The sky is the limit; you can be anything you want." She acknowledged that her response was not a misunderstanding of my question. Rather, she explained that she interpreted the question about messages as a math learner in that way because her high school placement had been aligned with career goals as well as aptitude. Looking back, she described her career goals during elementary school as "I want[ed] to be a pilot, nurse, baker, etc." indicating that she had absorbed the message that she could be anything she wanted to be at that time.

Identity Development During Middle School: A Time of Transition

Sarah's middle school years were a time of transition. She began to experience changes in math instruction, mathematical content, and primary place of learning. At the same time, the messages she received from others about her career potential began to evolve. Sarah recalled that her parents felt that they had given her a good foundation during elementary school and expected her to learn independently after that:

> My parents were very hands on in elementary school, and then as we got older it was more of the higher education was always an expectation, so they just knew, "She's gonna do what she needs to. We gave her that foundation." That was that.

At that same time that the nature of the in-school messages she was receiving began to change, these messages also became more significant because her primary place of learning was now in-school. She remembered that in elementary school she got gold stars and stickers and felt proud. But in middle school she started to “see red everywhere” which was very “eye-opening.” She realized: “I’m not good at everything.”

In-school instruction became more passive: “You were just being told to sit in a desk and follow along with the teacher.” Math was no longer fun to do in school. Sarah also felt that the math itself became more difficult during middle school, especially when algebra was introduced. She did not see the connection between arithmetic and algebra. She did not find the teacher to be of much help:

> I remember when letters were first introduced and I'm like, "What does X have to do with math? What is this?" I remember asking my teacher about it and she's like, "Well, soon you're gonna see A through Z, so you might as well just get used to it," and I was like, "Okay. Great. I guess letters are now a thing." And then letters and fractions became a thing, and I was still stuck on the letters there.

Sarah’s career-related identity and identity as a math learner were always strongly connected and began to shift in response to messages she received. The career-related messages she received also began to transition both in-school and out-of-school. The in-school messages she received changed from “You can be anything you want to be” to messages that included the possibility that ability was a factor: “What are you good at and have a passion for.” Internally, she was uncertain about her career choice, but was considering nursing: “I don’t really know what I want to do. Nursing seems okay.” She remembered that messages from home continued to include: “The sky is the limit you can be anything you want” However, the fact that she was

considering nursing may have marked the beginning of her parents trying to influence her career choices. Their push for her to go into the medical field had not yet become explicit and she might not have recognized what was happening.

Sarah also received messages about herself that were related to her status as an English language learner. She described her home as tri-lingual during her elementary school years. It was not unusual to transition between French, English and her native language. Sometimes she would come to school speaking French. Although culture was very important to her family, there came a point at which her parents were persuaded by school staff to speak English at home to help her improve her English. She also received help learning English through 7th grade through her school districts' English as a Second Language (ESL) program. Support included help with writing cursive letters, reading and personal in-class help during English class. She was tested every three months and by middle school, she typically tested two years above grade level in history and English. Sarah felt that she was fully fluent in English by 7th grade.

Identity Development During High School: Messages of Limited Possibilities and Potential

Early in the interview, I asked Sarah if there were any elementary school teachers that stood out to her and made a difference in her learning. She told me that she did not remember any particular teachers until 8th grade. Without prompting, she went on to tell me about each of the math teachers she had from 8th grade through 11th grade, collectively describing them as "all horrible." Sarah spoke for more than 12 minutes, without prompting or interruption, giving a critical evaluation of each of these teachers. She did not provide an assessment of her 12th grade teachers because she did not consider the classes she took that year to fulfill her math requirement to actually be math courses. Instead of continuing on to pre-calculus, she took a semester of marketing and a semester of personal finance, which "gave me my math credit and

there was barely any math involved." She had decided that as a senior: "I'm gonna give myself a break. I'm tired of math."

Sarah described her 8th grade teacher as "probably the worst teacher I think I've ever had. She never interacted with the class. She described her 9th grade Algebra I teacher as very nice, but lazy and as "another really bad teacher" who was fired at the end of that school year. Her 10th grade Geometry teacher also lectured without interacting with students; although that teacher did have some suggestions for strategies to use while doing homework such as looking at math related websites. At the time, Sarah felt the teacher's suggestion was not appropriate and that the teacher should have provided better instruction instead. Later on, however, Sarah concluded that this could be a useful learning strategy. Sarah took Algebra II the following year in 11th grade. Again, she was highly critical of the teacher and her teaching style. But this teacher "was very interactive. She always walked around" and always asked if anyone needed help. On the surface, this seemed like a contradiction. This teacher did what Sarah told me she wanted her previous teachers to do.

Sarah also described having experienced microaggressions from math teachers as well as support staff and counselors. When her earlier criticisms of teachers are considered together with these microaggressions, they revealed that something much deeper was going on. She referred to a "form" that followed her from year to year which stated that she was an English language learner and had a hearing disability. She found teachers' reactions to this information offensive. She found their condescending tone to be hurtful:

> I don't know. I hate feeling like I'm stupid 'cause I know I'm not. But with me being in ESL and I've always had extra help, it's just sometimes ... 'Cause you always see on a student's form, "She's an English learner student. [Sarah] has a hearing disability." So, teachers or tutors had the tendency to talk to me like I was stupid. "I don't need you to speak slower. I don't need you know. Just talk to me like I'm a person."

Sarah would often turn to her friends for help. She found it easier to ask them because they did not use a condescending tone. There was no need to keep her guard up against an implicit message of inferior intelligence:

> It was an odd thing. It was easier to ask my friends for help 'cause obviously they don't talk to me like, "This is what you do now, [Sarah]." They're like, "Okay, [Sarah]. This is what you do. Then you have to carry X and do this." It's like, okay. My friends are cool, but professional help in the sense of a teacher or a tutor, it was always like they talked to me like ... they just treated me like I was dumb, I guess is the easiest way to explain it.

It was often the case that her friends felt confused after math class as well. She found it helpful to work together for understanding: “We would piece it together at lunch, like, ‘Okay. You got this part. I understand this part. She understands the middle. Let's bring it together.’”

Although Sarah found some comfort while working with her friends, feeling that she did not really understand the material led to anxiety during math tests: “I always felt confused and then when tests came, it just turned into a feeling of anxiousness. I would immediately blank out.” She would instruct herself to do as much as she could for partial credit which usually added up to a passing grade even though she did not feel confident in her understanding. At her high school, 50 points out of 100 was enough for a C in math. She felt that she was “just coasting by each class.” She continued this pattern of passing while only knowing 50% of the content throughout her high school math classes: “Then I'd be onto the next class and it repeated itself. I was like, ‘Okay. I know something**.** I may not understand it, but I know enough to get half.’” Her elementary school goal of doing well enough in math to move on to the next grade carried over to high school: “Even if I had all As and then that one C, I was like, ‘I’m still moving on. It's okay.’”

Sarah received negative in-school messages about her overall academic potential from high school counselors. The final section of the background questionnaire refers to the social

constructs of race/racism, gender, class and disability status with a follow-up question: “Has your opportunity to learn been impacted in any way based on any of these or other constructs”? Sarah wrote:

> While I was in high school, I did take AP/Honor classes for other subjects. Math just wasn’t my thing. Often times in these AP classes I’d be the only black student. In my AP Biology class first semester there were only 3 of us. I was the only girl. Second semester I was transferred into a different class and was the only person of color in that class. I have a wide range of friends in terms of ethnicity. When it comes time for us to register for our classes for the following school year, we’d notice that our counselors would often recommend that we shouldn’t take certain classes even if we were recommended for it. I remember one of my Hispanic friends were upset because her counselor refused to recommend her our schools engineering program (despite her meeting the requirements) and said that she should take something less challenging like food prep.

Sarah explained that she had started taking AP classes as a sophomore in high school and that counselors consistently tried to discourage her, questioning if those were the right classes for her: “Are you sure that AP biology's a good fit for you? This seems like a lot to do.” Even after showing the counselors that she had both her parents’ permission and the teachers’ permission to take these courses, counselors would continue to push back: “Just because you shouldn't overwork yourself or overload yourself," which Sarah felt was insincere: “It was like that kind of odd, fake concern.”

Sarah’s profile sketch entries reflected the messages about career potential that she heard in-school which evolved from “What are you good at and have a passion for” during middle school to “Passion is important but realistically what can you do?” during her high school years. The messages Sarah received from her parents about potential career choices became stronger and more direct during high school. On her high school profile sketch, she wrote that she received the out-of-school message: “You should be this field; anything else is a waste of time.” She explained that her parents are immigrants who came to the U.S. in their late 20s. Although they were both educated, they needed to change career fields to find jobs. For, example, her

father had been a political science major in college but struggled to find a job in the U.S. He went into nursing because it was a rising field. Her mother had been a geology major and wanted to be a teacher in the U.S. But Sarah's father discouraged her from doing so because of her accent: "Those kids are gonna pick you apart." Her mother is currently in school for a Bachelor of Science in Nursing. Her parents concern about Sarah's career options stemmed from these experiences and from a feeling that Sarah shares with them: "Immigrants really aren't respected in this country."

Strategies for Self-Protection: Rejecting Expectations

Sarah protected herself from what others expected of her in three ways: (1) She rejected messages of low ability that she received from math teachers and tutors about her "low ability;" (2) she rejected messages from counselors that, overall, her potential was limited; (3) and she partially rejected messages from her parents that limited her career options. These messages that she received from her parents about her career options were mixed. Her parents believed that she had the capacity to excel both academically and professionally, but they also wanted to limit her career choices out of concern for how others would treat her if she chose another path. She accepted that she could do well professionally but wanted to choose her own path.

Sarah remembered that each of her 9th, 10th, and 11th grade teachers were willing to spend extra time with her, but when she did ask for help, she always left feeling more confused. Their condescending tone was a distraction from learning. She rejected their negative messages internally and used nonverbal cues to get her point across to them:

> Then I would find like, "Okay, she doesn't know English. It's not her first language." Like, I'm good. I promise you; I understand what you're saying. You don't need to speak ... I know I have a hearing problem, but you don't need to be like, "This is what you do." I would go for tutoring in my freshman year and my sophomore year and they would see that, and they would act like I was dumb. I would look at them like, "What are you doing? I've been in this country for a good 10 years now. I promise you; I know English.

> Why are you speaking slower?" And I would leave thinking, "Do they do that to every other student? Or is it just me?"

Sarah told me that she is a very prideful person, as many of her family members are. This combined with the way that she had been treated, led to her being reluctant to ask for help during 9th and 10th grade. But by her junior year of high school, Sarah concluded that needing help is not shameful and has felt motivated to ask for help as a necessary part of academic success:

> I always wasn't the best at asking for help and now I'm in a mindset now where there's no shame if you need extra help, but when I was in high school it's like, "I don't need anyone's help. I can figure this out." Then obviously that kinda changed by my junior year where it's like, "I need good grades. Can you please help me?"

Sarah also protected herself from negative in-school messages from her counselors who doubted that she could be successful in challenging coursework. She did so by not valuing their beliefs: "I didn't really care about their opinions." To back-up her assessment that their opinions were not valid, she pointed out the high turnover rate in the counseling department: "Their opinion didn't really matter just because they were a revolving door. There was a new person every three months.

Sarah protected herself from her parents' attempts to control her career choices by choosing a profession outside of medicine. She understood that her parents' choice was based on their own experiences of feeling disrespected as immigrants in this country. But Sarah felt that the choice should be her own. She felt that her parents were being too restrictive and overreaching. They were "pushing the medical field down my throat." She remembers feeling that she was risking rejection by not following their plan for her:

> "You need to become a doctor. We don't want an engineer; we don't want a teacher. We want a doctor." I was just like, "I wanna go to law school." So, I'm the black sheep of the family. Granted, a lot of my cousins aren't going into medicine; like one of my cousins is becoming a social worker. Hello, come join the club of the rejects.

Sarah's profile sketch entry about messages she told herself during high school reflected her intention to choose her own career path rather than follow her parents' expectations: "I'll do what I'm passionate about and makes me money." She reflected on her earlier thoughts of becoming a nurse and realized that she had only considered it to please her parents and fulfill their expectations of her. But she eventually realized that that was not what she wants to do with her life:

> But at some point, I was like, "I'm gonna become a nurse or I'm gonna become a surgeon or I'm gonna do something like that." Then I realized I'm really trying to please my parents and I just like law. I like writing contracts, which is weird. So, it's like, "I'm gonna go to law school and this is what I wanna do with my life," and my parents were like, "No doctor? No nurse?" She's like, "At least become a nurse. Please become a nurse." This is me; "I'm gonna go to law school." So, we're still working on that. I told my parents I would do medical malpractice and they're like, "Something."

Sarah's law school choice helped her to reject two types of identities that had been designated for her. By choosing a career that required not only a college degree, but a professional degree she was able to reject messages of low expectations and unfounded views of low ability she had received from teachers and school staff. This choice also allowed her to reject the occupations selected for her by her parents.

Brad: "Environment is a big part of learning"

Brad felt alone in learning and had no one to help him through his many difficult transitions in learning environments. Brad attended both predominantly Black and predominantly White schools, alternating between the two during elementary, middle and high school. It came across to me during the interview that Brad needed for me to understand what happens inside predominantly Black schools and how learning is impacted by the environment, with lasting consequences. Brad didn't just change schools; he experienced significantly different learning environments. Brad could have felt bitter or angry about the abusive learning environments he

has endured. Instead, he told me with a gentle voice that he wants to tell his story and to try to change the system for the benefit of others.

Brad identifies as a Black male and was 21 years of age at the time of the interview. He speaks English as a first language and has no physical or learning disability. Although he is not first in his family to attend college, he did not grow-up with college-educated parents; his mother went to college while he was in high school. His first college experience was cut short almost immediately because financial aid did not come through. He qualified for and received financial aid at Midwestern Community College.

Brad attended all public schools. The neighboring predominantly Black and predominantly White school districts he attended share a border but little else. Brad had trouble remembering the specific chronology of schools, but he remembered how he felt within each environment. The first two schools that he remembers attending were located in the same large, middle to high socioeconomic status, predominantly White school district. At some point, possibly after 2nd grade, he left there and went to an elementary school in a low socioeconomic status, predominantly Black district, then returned to the predominantly White school, possibly at the start of 5th grade. Brad attended a predominantly Black middle school in 6th grade, and a predominantly White middle school for 7th and 8th grade. He started high school in the predominantly White district and attended 10th and 11th grade in the predominantly Black district. For a short time, he attended an early college program, but found that it was too difficult. He now had the option of returning to the predominantly White school for 12th grade and did so.

At one point, Brad began to tell me *why* he changed schools so frequently, paused, and moved on. He had already generously shared many painful memories of his K-12 experiences with me. So, out of respect for his privacy, I followed his lead and decided not to pursue a

dialogue about it. What is more important than the reasons for his school changes is the way that he experienced the changes—the way that he felt, the ways that he was impacted as a learner and as a person, and his message about the toxic environments that many students experience and have a lasting impact on learning. It is also important to consider the ways that he was impacted by messages he received about school resources and safety in predominantly Black schools.

Impact of Learning Environment

Brad's specific memories of his schooling experiences are limited and scattered due to frequent changes in his learning environments. But he did learn, in a memorable way, that "the environment is a big part of learning." From Brad's point of view, the school environment includes much more than academics; it also includes the way that students are treated by authority figures, a sense of belonging, peer influence and engagement, as well as resources that include teacher quality, and school safety. In particular, Brad remembered that students are treated differently in predominantly Black schools than in predominantly White schools in all aspects of the environment. He also remembers how he felt and that his feelings carried over from one institution type to another and have had lasting consequences for his life and learning.

Alternating between predominantly Black and predominantly White schools imposed on him a deep awareness of the differences between the two. Brad explained that "just being in a predominantly Black school is different, it's just different than White. It's just how it is. It's crazy." He continues to be acutely aware that his direct experiences with inequities in education provides him with a unique vantage point for understanding and describing differences between the two school districts that he attended.

Predominantly Black School Environment

Brad's memories of specific experiences at the predominantly Black elementary school that he attended are blurred, but what he does remember is that when he returned to a predominantly White school, possibly in 5th grade, he was behind his classmates in learning. He then moved on to a predominantly Black middle school during 6th grade. His 6th grade memories are even more blurred, although he remembers this middle school environment as one in which he felt that he "didn't belong." He noticed that he had been better educated than his peers due to a difference in 5th grade learning opportunities. As he described it, "I felt like the brainiac 'cause they didn't have the education that I had." He also said that he became a "lazy" student that year because he already knew everything being taught: "I don't remember them teaching me anything, seriously. It was bad."

Brad received messages about the different ways that students are value. One type of message was in the form of school resources. At the start of the interview, I asked Brad about his earliest memories of learning math. It surprised me that he not only included but repeated "we had our own math books." The importance of this statement became clear after he described school resources at the high school level. Students were not provided with printed worksheets or other handouts at the predominantly Black high school. Nor did the predominantly Black schools have the technology that the predominantly White schools had. When Brad was at the predominantly Black high school, students were given Chromebooks, but students didn't know how to use them and didn't know how to type, which slowed learning rather than enhancing it. Brad described unqualified teachers including "a Spanish teacher who doesn't know Spanish."

Brad had all Black teachers for the first time in 10th grade, but he was disappointed because "they weren't teaching me anything." In 11th grade, he had both Black and White

teachers, but they were new and “getting used to it.” Instruction time was limited due to a combination of inexperienced teachers and disruptive students:

> Man, I remember it was this young teacher, she was young, and I don't think she knew how to handle the class. She was new and kids were so bad. We were so bad, so half of the class was basically stopping interruptions. That's what I remember; stopping interruptions and I'm just looking at the paper, just filling it out already 'cause I know how to do it. It was just easy stuff, so while she, "Can you guys stop? Can you guys stop talking," I'm just doing it. I already knew how to do it.

According to Brad, the lower level of academic rigor was more consequential in high school than in earlier grades. After failing 9th grade math at the predominantly White school, Brad was initially placed in a remedial class in 10th grade at the predominantly Black school. But he felt like he had gone down a grade. Even though he had not learned enough to pass Algebra I in 9th grade, he stated that he already knew the content of his initial 10th grade class. As this became apparent to his teachers, they changed his placement.

School attendance was inconsistent among Brad’s peers; classrooms felt empty at the start of each school year, then became crowded. Second semester, classes were empty again. Skipping classes was common among his peers. But, crowded or empty, the classroom was a toxic place for Brad. He experienced within-race racism as students, and even teachers, mocked him and called him “White.” After Brad’s placement change in 10th grade, he was in class with older students which posed a threat to his safety:

> I was kinda shy back then. I was kinda skinny. I was skinny kid so I wear the same jacket every day so nobody would see my bones, but you gotta understand, remember I said I was in the 10th grade, so I’m about 16 in classes with 18-year-olds, 19-year-olds, 17-year-olds. They all bigger, but they all cool. They'll talk about you and everybody laugh; the teacher don't say nothing. The teachers don't say nothing! I'm telling you, they make fun of you too.

In addition to not being provided with challenging academic work, Brad’s ability to learn was negatively impacted by the toxic atmosphere and anxiety over his safety. He attributed his

blurred memory of high school to the harsh environment: “I was in a blur because I wanted to get out of it.” Brad had no friends at school but met two students who helped to protect him from fights. Brad did not want to fight, but as he explained: “I had to protect myself … I fought a little bit, I *had* to. People just hate for no reason.”

He described the way that students are regarded at the predominantly Black high school: “They treat us like trash. It's just that simple.” He described a prison-like atmosphere at the school: “It felt like prison down there. That’s what it felt like. … even the classrooms, no windows, stuff like that.” Brad explained that people who do not experience this environment for themselves cannot fully understand it the way that he does:

> But I've seen it, so I understand why these people drive by and why people turn out the way that they do: "Why did he come out like that? Why is he dead? Why is he in jail? Why is he like this?

Brad recalled a particular incident that highlighted the complexity of his feelings which included compassion as well as outrage. He explained that as he tried to get up from his desk, a girl blocked him and wouldn’t let him get up. Not wanting to fight her, he “just held her until she calmed down.” They were both suspended for two weeks. As he told this first part of the story, his tone was that of compassion for this girl, explaining that “I guess she was feeling some type of way that day. She just wanted to pick on somebody; she came to pick on me.” But the story does not end there. Brad’s tone changed as he described the way that the security guard abused her authority. The day that Brad and this girl returned to school after the suspension, the security guard mocked him in front of her trying to instigate further trouble and added: “She whopped your ass, didn’t she?” At the time of this incident, Brad was feeling fed up and didn’t want to care about school; he didn’t want to be there. He felt relieved about the suspension: “That was the best two weeks of my life. … I was happy to get suspended.”

Predominantly White School Environment

Brad began his schooling in a predominantly White district. His earliest memories of learning mathematics begin there. They include working on multiplication charts, taking arithmetic tests, using flash cards, and playing math games to gain speed. At some point, he went to school in the predominantly Black district but has no particular memories of it. Changing schools meant coming "in on different lessons." When he returned to the predominantly White school in 5th grade, teachers initially thought he was of low ability and placed him with students who struggled, but as they became aware of his ability, his placement was changed. Homework was now required; he is not sure why, but he typically didn't do it. His home was a busy place and not conducive to doing homework: "It may be because of the family. I have 6 brothers and 3 sisters, so it was a lot going on. It's just probably what is was." He began to get low grades, but he did not feel those grades were a correct reflection of his ability.

Brad felt the predominantly White schools were a better fit for him both academically and socially; he had a greater sense of belonging there. The predominantly White schools had higher level, more challenging academics but Brad did not feel supported in learning and was often unable to take advantage of the higher level of academics. He was given bad grades rather than help. He described learning fractions as especially problematic. He was already behind and had no one to help him "so I didn't pay attention 'cause it was too hard." As he explained it:

> Nobody helped me. That's the problem. If I had help, I would've been so good at it, but I didn't have help. I just threw it to the side and they just going, "Oh, he don't know what he's doing, so just book him. Give him a bad grade on his report card 'cause he don't know," but they didn't help me. That's the problem.

But the transition in 5th grade went beyond academics. At the start of the school year, Brad didn't have any friends in his class. He wanted to fit in, so he relied on his sense of humor and ability to make people laugh to help him through the social side of the school transition:

> Goofing off, making people laugh. That's what I like to do. I can make somebody laugh. I would make the whole class laugh. Yeah, that's what I did 'cause everybody knew me at one point, you know? That's kinda what I wanted though, 'cause I didn't have friends. I made people laugh.

As a result of not doing homework and being funny in class, teachers and classmates began treating him "like the class clown." Although he enjoyed making people laugh, he didn't like having his intelligence underestimated by teachers and peers who treated like a "dummy." Mid-year in 5th grade, he decided to prove himself by studying for a spelling test and received 100%. He felt proud when the teacher acknowledged his accomplishment. He also recalled a time later that year when he surprised the class during a multiplication game when he was the first to correctly answer a problem:

> I got it right first, and the whole class was surprised. Like everybody was like, "What?!" They was like, "Oh my God, oh my God." But in my head, I knew I knew it. I was like, "Why you all so surprised? I know this stuff," but you know what I'm saying? That's why I'm saying they kinda treat me like a class clown dummy. I didn't like that.

After attending 6th grade at the predominantly Black district, Brad returned to the predominantly White district for 7th and 8th grades. He had a sense of belonging there both academically and socially. He had friends and family at that school. When compared to the predominantly Black schools, he felt the predominantly White school was his "level of learning," but still did not feel challenged. However, he was behind his peers as a result of not having the exposure to academics that they had. As he began to realize this, he also began to notice differences in schooling between predominantly Black and predominantly White schools. He continued "goofing off," but this time he was not trying to entertain the class; he felt that he had become "lazy" while he was in the predominantly Black district and it carried over to 7th and 8th grade. He enjoyed having friends and family around, but he did not receive any academic support from them or from teachers and felt alone in learning.

Brad attended high school in the predominantly White district during both 9th grade and 12th grade. The missed learning from changing schools led him to goof off and do very little homework each time he returned. As a result of missed learning and not applying himself, as noted earlier, he said that he failed his 9th grade math class. Before returning to the same high school for 12th grade, Brad briefly attended an early college program. He was proud of the fact that he applied for the program, passed the test and was accepted. But after two years at the predominantly Black high school, he felt underprepared and that it would be too difficult for him. At this point, his alternative was to return to the predominantly White high school for 12th grade where he could play football as well as take more appropriate classes and did so.

School resources were noticeably better at the predominantly White schools. There were no safety concerns; no prison-like feel. He received printed worksheets and packets for studying. His peers were planning on going to college and he had a girlfriend who pushed him to work harder in his second semester 12th grade math class.

Brad felt both underprepared and unchallenged in 9th grade and the first semester of 12th grade. His intelligence was often underestimated and too little was expected of him by those who did not understand the role of his past learning and negative impact of misaligned curriculum and rigor between the two districts. He was behind in learning each time he returned to the predominantly White district.

But in 12th grade, Brad learned that he needed to take both 11th and 12th grade classes that year because the predominantly White schools had higher standards for graduation and some of his credits from the predominantly Black district did not transfer. This left him in the position that he had to pass every class to be able to graduate with his peers, including Algebra II. His experiences in math that year were vastly different between first and second semester. The

learning that he missed out on at the predominantly Black schools was especially problematic during his first semester. He felt discouraged and that cheating was his only option:

> Then 12th grade I remember math. The first semester of math in 12th grade, I cheated and cheated the whole class 'cause I didn't know coming from [the predominantly Black to the predominantly White school] for 2 years, I didn't know most of the stuff. I was like, "What is this?" So, I cheated the first semester.

Second semester, Brad had a local college professor for math. This professor had a reputation in the school for making students take his class seriously and work hard. Brad was inspired by the professor who had high expectations and taught well:

> Then the second semester we had a college professor; I'll never forget him. He made us work. Like, he *made* us work and that's when I kinda applied myself and I knew it. He actually taught us, so I learned, and I studied it 'cause I knew that if I failed this class, I was gonna fail 12th grade. So, I had to learn.

Brad explained that there were a lot of students in that class who typically goofed off in class. But when they didn't pay attention in this class, the professor would call on them and send them to the board to do a problem:

> It was crazy to see that though, 'cause it was a lot of people that played around. Even me and my girlfriend was in there together, but we couldn't do anything 'cause he was there. Then we knew how much the tests were worth, but I don't know what they were thinking. I was just trying to pass the 12th grade.

Brad told me repeatedly that the 12th grade professor taught him more than any previous teacher. Brad appreciated the professor's no-nonsense demeanor and high expectations: "He taught me the most. That's where I learned the most math at right there. Yeah, he made us pay attention for sure, somehow." Brad needed 60% to pass that class and to have enough credits to graduate with his peers. He achieved 58% on the last class, received a failing grade, and had to take an online math class over the summer so that could graduate. He was not able to walk across the stage with his peers and got his diploma in the mail. Although Brad did put forth more effort than usual, the professor saw that he could have worked harder. Brad remembers that the

professor told him: “You just didn’t apply yourself enough.” Brad could have been resentful that the professor did not give him a passing grade when he was so close, but instead he was grateful for the good teaching and especially for the high expectations and acknowledgement that failing the class was not due to lack of ability.

Development of Brad’s Identity as a Math Learner

Messages Received

Brad received many implicit and indirect messages about himself as a math learner. Throughout his primary and secondary education, Brad noticed differences in the learning opportunities provided to students at predominantly Black versus predominantly White schools. He repeatedly stated that he felt like he was going down a grade each time he switched from a predominantly White to predominantly Black school and was behind in learning when he returned to a predominantly White school. Each time he returned to predominantly Black schools he saw that his peers were less educated than he was because they didn’t have the opportunities that he had just received. When I asked him about messages he received in middle school, he told me that he was not taught anything there, which implied a negative message about how Black students are valued as learners. Students at his predominantly White schools had challenging curricula; they had their own textbooks, handouts and work packets; they had qualified and experienced teachers. Students at his predominantly Black schools had remedial curricula; they had few textbooks, handouts or work packets; and they had unqualified and inexperienced teachers.

Brad also received messages about his value as a result of the safety issues at his predominantly Black schools. As described in the previous section, a security guard whose job was to protect students tried to instigate a fight. Teachers are also supposed to keep students safe,

but at times joined immature students in mocking him. Brad described an environment filled with hate and in which students were treated "like trash." Brad expressed no memories of feeling physically unsafe at the predominantly White school.

Brad received implicit messages in predominantly White schools in the form of being underestimated by teachers as well as peers until his final semester. Each time that he changed back to a predominantly White school, he was behind in learning, but he did not feel previous lack of opportunity was recognized. Instead, peers as well as teachers treated him as less intelligent than he knew he was. He received a message when his 5th grade peers treated like a "dummy" and when he felt that his 8th grade teacher "thought I was really stupid." But he also received a message when his 5th grade teacher acknowledged his accomplishment after he studied and received 100% on a spelling test. During his final semester he received an indirect message when his girlfriend indicated that she believed he could do well when she pushed him to work harder. He received a more direct message from the college professor who told him "You just didn't apply yourself enough," and by not giving him an inflated grade because he knew he had the ability to do better.

Messages from home were limited and indirect. He felt unsupported in learning and does not remember messages from family during elementary or middle school. But when he was in high school, his mother went to college. She gave him encouraging advice about college, which indirectly gave him the message that she felt he could go as well.

Self-assessment and interpretation of messages

Brad's identity developed throughout his time in each environment. Messages he received were overwhelmingly negative. When he was in the predominantly White school district, his ability was misjudged as inferior. When he was in the predominantly Black school

district, he received messages about how Black students are not valued in a society that allows predominantly Black schools to be so toxic. His own self-assessment of his ability, that he could have learned had he been given the opportunity was always consistent even as it was in conflict with messages from each environment.

When I asked him about messages to himself in elementary school, he responded by describing self-instruction, which fits with later statements of feeling alone in learning and unsupported both in-school and at home. Brad had to rely heavily on himself, starting at an early age. During his elementary school years, he watched public television after school and learned basic arithmetic. He accepted the message from these educational programs that he could solve problems more than one way.

Brad always maintained that he was and continues to be "capable", "smart" and even "super smart." He knew that when he was given the opportunity and applied himself, he could learn. He rejected messages of low ability associated with low grades. He remembers that he was given bad grades when what he really needed was help and rejected messages from teachers and peers that he is a "dummy" or "stupid." For example, he believed that his difficulty with fractions was due to not receiving needed help.

He understood that changing learning environments was a huge impediment to his learning. Yet, he often referred to himself as "lazy." When he was given an opportunity to learn in a meaningful way and held to a high standard during his last semester of high school, he began to apply himself. One semester was not enough for him to turn things around and apply himself fully and he did not quite achieve a passing grade. But he felt that when he was taught the most, he learned the most. His reaction here showed that he valued learning over grades.

Strategies for Self-Protection: Multiple Strategies

Brad's need for self-protection fell into two broad categories: physical protection and protection of his self-worth. To protect himself from physical harm, he kept his jacket on so that the other students would not see how "skinny" he was. But this was not always enough. He needed to have a couple of students who had his back, and although he did not want to, at times he had to fight to protect himself. This was perhaps a more immediate and more obvious form of self-protection. But perhaps less obvious was a need to protect his sense of self-worth. When he was new in school and didn't have any friends, he acted like the "class clown." When he was behind in learning, wasn't receiving appropriate instruction, and didn't know how to move forward in learning during the first semester of 12th grade, he cheated. Most of his push-back on negative messages was internal dialogue. During the interview he often referred to himself as lazy when describing how others were judging him as having low ability. When he had the college professor during the second half of 12th grade, he no longer had to put forth effort to protect his dignity and could instead begin to put effort into learning.

Concluding Thoughts

Near the end of the interview, I asked Brad what he felt were the greatest obstacles to his learning. His response surprised me. He began by talking about his own lack of effort, then described how he felt "super smart" after he did apply himself. Next, he expressed his frustration with being underestimated by his peers at the predominantly White schools and gave an example of a time that he proved himself in class. Perhaps he considered his lack of effort to be the greatest obstacle because it is something he can control, whereas he could not dismantle structural racism in schools or choose to stay in the predominantly White school district.

Brad told me that the wants others to know about his experiences and to understand that what goes on inside predominantly Black schools is connected to problems in society beyond our schools. He understands that those who have not experienced it do not understand or even know about it: “It's sad because a lot of people don't know it. A lot of people.” Even his own younger brother “won't understand it because he hasn't seen it, but I've seen it, so I understand why these people drive by.” Brad wants to promote change for the next generation, to: “figure out a way to change what I've been through 'cause I would definitely not have my kid go through that.”

Summaries of Non-Focal Participants

Victoria

Victoria described herself as “a numbers person” who feels strong with math and learns math with ease. When she was growing up, her uncle gave her math books as gifts and helped her to see math as a fun activity that she could be successful at. This set a positive tone for her future learning. However, during her first two years of high school while taking Algebra I and II, she was bullied by other students for doing well and participating in class. She felt like her peers did not take the class seriously which led to her not taking it as seriously as she otherwise would have. She also felt that her teachers did not care about their students. In part because of this, she chose to enroll in a regional technical high school for half of her school day during her junior and senior year. She took geometry and statistics at that school and felt that the students were more serious there. In addition, the math teacher, who she had both years, was more caring. She felt less anxiety, but the experience of the previous years had a lasting effect.

She also described having extreme test anxiety, with physical reactions such as “sweating bullets,” “shaking,” and feeling “frozen” during math tests, including standardized tests. She told me that she found multiple choice tests, including the math placement test, to be particularly

anxiety-inducing because no one can see her work and one small error means no credit for all of the work that she did. Time limits on tests also add to her stress.

Victoria's high level of math test anxiety seemed to me, at first, to be in contrast with her high level of confidence as a math learner. But she also described herself as an extreme extrovert who feels all alone during tests which she told me adds to her anxiety. As I considered her past experience with being bullied together with feeling alone during tests, this combination began to make sense to me as a possible explanation.

Furthermore, Victoria told me that she does not feel anxiety while using math outside of the classroom. At the time of the interview, Victoria was enrolled in college part-time while working full-time as a human resources coordinator. She reported that she regularly uses math at work which includes creating graphs as well as writing and inputting equations into Microsoft Excel. She indicated that she feels pride rather than anxiety when using her math skills in this way. She feels especially proud when her boss or co-workers ask for her help using math.

Kelly

As a student who was homeschooled throughout K-12, Kelly provided an interesting perspective on learning and assessment. Her mother was her teacher, her siblings were her classmates, and at times, her father helped with math. Her mother followed a particular faith-based homeschool program that included guidelines and resources such as textbooks, worksheets, and assignments. Kelly was encouraged to use online support such as Kahn Academy, as needed. While college degree requirements provide some incentive to learn mathematics, Kelly is motivated internally as well. She stated that she wants to be an educated person which, from her view, includes knowing mathematics.

Kelly feels that with hard work and dedication, she is able to do math and to be successful at it. At the same time, she feels that she is not good at math: “My whole life, I felt like, I’ve never been good at math.” Kelly remembered liking math during early elementary school but has disliked math ever since. Her dislike of math began when she began to struggle with the content. She sometimes felt defeated and frustrated. Her mother would tell her to take a break and come back to it. Her mother also encouraged her to persevere until she understood. Kelly eventually began to find math to be very rewarding as she came to understand more difficult topics.

As a homeschool student Kelly, did not receive grades throughout K-12. Topics were self-paced, and tests were repeated until she demonstrated that she understood the topics well enough to move on. Her mother used the homeschool program guidelines to determine her progress. The feedback she received from testing was limited to the identification of areas in need of improvement before moving on. During her high school years, she studied Pre-algebra, Algebra I, some Geometry, and very little Algebra II. No state-level testing was required. She did take the SAT during her last semester of high school because she planned on going to college, but she felt that it did not go well because she had to guess at many questions due to the time limit. As a college student, her perspective on grades is mixed. Grades provide deadlines which can lead to moving-on regardless of learning; but they also provide motivation and a sense of urgency that was missing in K-12. She also feels that grades on assignments as well as end-of-term grades provide valuable feedback and can act as rewards.

Mary

Identifying as someone who is very good at math has been a source of pride for Mary, beginning with her earliest memories of learning mathematics in elementary school; she

sometimes offered examples as proof. She told me that she loves math and that it was her favorite and best subject throughout K-12. At times, she was placed in slower-paced math classes, but she told me that she felt that the placements were a mistake and dismissed the potential messages that they may have carried. Although Mary speaks English as a first language, she told me that she has always struggled with it; she was required to take two English classes during 8th grade. She felt that she always did well in math and science but struggled in other subjects.

Mary expressed a strong preference for doing math her own way, the way that she considers to be the easiest way and stated that listening to alternative methods is confusing to her. She described one of her earliest memories of learning math as being taught "easy ways to remember how to count by 2, 3, 4 …," and describes doing math as "just following steps." She frequently expressed needing to be shown how to solve multiple examples of the same type of problems with detailed steps at a slow pace but all using the same solution method.

There were times when she did not do well in math. To deal with the pain of feeling unsuccessful, Mary shifted full responsibility for her difficulties learning to the teacher as an act of self-protection. Although she frequently described herself as good at math, she also told me that the only way she can do well is if the teacher makes it simple. She described feeling frustrated with teachers who "want you to learn their way.

Mary filled out the background questionnaire quite thoroughly, but she did not answer the question, "Have you been identified as having a disability?" She answered "No" to the follow-up questions indicating that she does not feel she has a disability and that it has not impacted her learning. During the interview she brought up the topic and alluded to a disability; but she said that it isn't really a disability, she just needs more time. Although she brought it up, she also

came across to me as sensitive about the possibility of having a disability, so, out of an abundance of caution, I did not press her on it.

Ashley

At the start of the interview, I asked Ashley to tell me what her earliest memory of math is. She responded: "That I was never good at it." Throughout the interview, Ashley consistently described herself as someone who is not good at math. But she also reported that she can do well in math, making it clear that her self-efficacy was only one part of her identity as a math learner. Ashley's memories of early learning math were mostly limited to learning at home with her grandmother. Help was always readily provided. She never had to ask. She received strong messages from her parents as well as her grandmother throughout K-12 that she can learn math, but not without additional help from home.

She was identified as having a learning disability and began receiving accommodations in either 3rd or 4th grade. She described her disability as making it harder for her to understand and as having trouble remembering. She remembers that after being diagnosed with a learning disability, she was placed in classes with an extra support teacher, who provided additional, individual support. She did well in those classes, even achieving 90 to 100% on all assessments during 11th and 12th grades. But even then, she did not feel that she was good at math. She told me that she did well because she "knew everything," but held on to her self-assessment that she is not good at math. She also described her experience in these classes as: "They spoon-fed me" and that she could "sit there and sometimes they'd let me use notes on tests."

This combination of readily receiving help while getting good grades strengthened her belief that she could learn and do well, but only with assistance. As a result, she does not allow herself to feel a sense of accomplishment when she does well. Ashley rejected encouraging

verbal messages from teachers that she could work independently. Instead, she constantly looked for reassurance on math problems her teachers felt she already knew how to do and could do on her own.

In response to my first question, "What is your earliest memory learning math?" Ashley exclaimed "That I was never good at it." I felt that she was trying to prompt a reaction from me. She succeeded; I made an unintentional gasp. She looked quite pleased. There are places throughout the interview where Ashley talked about making statements such as this to her teachers whose response was to reassure her that she is smart. I did not understand this until I considered the possibility that she was protecting herself from judgment by judging herself first and provoking others to reassure her.

Pearl

Having a learning disability is central to Pearl's identity both in general and as a math learner. She feels that she cannot learn math without a lot of help, and she expressed overwhelming concern about getting enough help. But she also described having only limited anxiety associated with math when she does have support. She reported that she feels confident doing addition and subtraction but not multiplication or pre-algebra. Pearl is motivated by the pride that she takes in her persistence and willingness to work hard.

From 5th through 12th grade, she had what she described as high-quality teaching and one-on-one in-school support, as well as support at home. Pearl started her schooling at a public school where she had a difficult time. She and her parents felt her needs were not being taking seriously. So, her parents enrolled her in a private school that specializes in helping students understand their own needs and develop lasting confidence and motivation. The private school's mission included preparing students for return to public school. Pearl's parents could barely

afford the tuition and after just a year and a half, she returned to public school. However, it was not the same public school. While she was attending the private school, her parents moved to a different school district. She described her experience in the second school district to be totally different from the first. Class sizes for students with special learning needs were limited to 12 students. She received in-class as well as homework support. Because of her disability, she was allowed to go at her own pace and was not required to take the usual math requirement for high school graduation. She worked on arithmetic through 11th grade and had some Pre-algebra during 12th grade but returned to a review of arithmetic.

Elijah

Elijah's family moved a lot due to his parents' jobs throughout his K-12 years. His memory of changing schools during his elementary years is "blurred." Early in the interview, he stated that curricular misalignment was not a problem because he had strong academic support from home. But later in the interview when we talked about his high school experience, he stated that moving was "terrible" and that a mid-year change in 9th grade was particularly difficult and hurt his grades. A second high school change between 10th and 11th grades led to a problem with credit transfer and had a negative impact on his math learning, in particular. Problems at home that started in 7th grade, which he only referred to as "personal issues," significantly impacted his learning from 8th grade through the end of high school.

Messages Elijah heard about himself as a math learner in-school and out-of-school throughout K-12, while not identical, were never in conflict. There are no indications that he rejected any messages. During his early elementary school years, he heard in school that he "catches on quickly," that he is "the best," and that he should "work on timed arithmetic tables." At home, he heard that he needed to study with his parents after school and that he is a "hands-

on-learner" meaning he benefits from examples. Elijah enjoyed playing games and winning prizes in math class during elementary school but there is no indication that he ever enjoyed math itself. Toward the end of elementary school, he heard that he needed help at home, and he agreed.

When I asked him what his earliest memories are of learning math, his response started with memories of doing arithmetic tables, but he went on to say that middle school math was harder and high school was "terrible." Middle school teachers told him that he is a good student who needs extra help with math. Elijah was encouraged by both teachers and parents to study and ask for help. He internalized these messages and concluded that he "was terrible" at math. He was determined not to fail and regularly stayed for help after school. Messages from school during his high school years were not as strong as before, but he had strong support from teachers who were willing to work with him. His mother continued to encourage him to get help at school and pushed him to learn. But personal issues noted above overshadowed the messages he received. Although he acknowledged that problems at home negatively impacted his math learning, he attributed his difficulties with math to himself. He describes a dichotomy of math learners and English learners: "there's math, then there's English people … I'm an English person." But hard work does matter: "If you dedicate yourself, you can be a math learner."

The "personal issues" that he experienced during middle school and high school combined with frequent school changes continued to negatively impact him after high school graduation. "I definitely internalized everything, and it eventually came out." He described himself at that time as a "hot mess," and that it "went on into college." He attended and dropped out of college three times. At one point, he joined the military and had to go on a mission, which is the reason for leaving college the third time. At the time of the interview, he was 25 years old.

He was enrolled in college for the fourth time, and on track to graduate within a year. His expected community college graduation date is eight years after his first college enrollment. His eventual goal is to transfer to an Historically Black College. Elijah spoke with a strong tone of determination as he described his current goals.

Summary of Findings

Each focal participant and nearly every non-focal participant described strategies for self-protection that they had developed during their K-12 years. The range of strategies they described are related to the range of experienced they have had. Each participant was influenced by internal as well as external messages they received as their identity as a math learner developed. Each participant was significantly impacted by the environment in which their learning took place. Those who received accommodations during K-12 reported a range of experiences in the ways that their accommodations were provided as well as the ways that they were impacted by those experiences. I found that my participants showed a high level of persistence in overcoming obstacles and prevailed in ways that are not acknowledged in the literature.

In Chapter 5, I discuss the significance of each of these findings as they relate to the questions raised in this study. I did not explore what my participants had to say about mathematics itself in the individual portraits or summaries because I felt that, in this case, their collective experiences were more meaningful than their individual responses. This includes what they described as being hardest about learning mathematics. I include this in Chapter 5. I also return to some key questions raised by Sfard and Prusak (2005) that relate to my study and are central to the identity portion of my conceptual framework.

Chapter 5 : Discussion of Findings

In this chapter, I discuss my findings and their relevance to teaching and learning in developmental mathematics courses. I begin by re-situating the reader with respect to my goals, research question, analytic questions, conceptual framework and central identity framework questions. Next, I discuss each theme that I found during analysis. Then I discuss my findings around the themes that I noticed during portrait analysis. I also discuss how my participants described mathematics and what they found to be hardest about it. Finally, I address the questions that were raised by Sfard and Prusak (2005) and relate them to my initial motivation for this study and to my research question.

Re-Situating the Reader

The purpose of my research study was to develop an understanding of who developmental mathematics students are as math learners in the context of their past learning and life experiences. My goals also included learning about the mathematical identities, beliefs, and emotions that my sample participants brought with them to their developmental mathematics courses and how these aspects of mathematical affect developed over time and influenced their actions. I wanted to understand how prior opportunities and barriers impacted their learning and identities. It was important to me to explore these issues using the students' own voices, to present their points of view, because I wanted to bring a new perspective to the research on developmental mathematics. At the same time, I also had the personal goal of being able to use the findings of this dissertation to improve teaching in developmental mathematics courses, including my own.

To accomplish these goals, I asked: What do developmental mathematics students' accounts of their past learning and life experiences reveal about them as math learners that their grades do not? I further developed analytic questions to unpack my research question and guide my study design and analysis:

(a) What identities did a sample of DM students develop as math learners prior to their enrollment in DM as reflected in their narratives?

(b) How did these identities shape their actions as math learners prior to enrollment in DM?

(c) What opportunities and barriers to learning did these DM students report experiencing, in school and out-of-school, prior to enrollment in DM?

(d) In their perspectives, how was their learning of math in K-12 shaped by such learning barriers and opportunities? How important did they consider this to be?

(e) What did these students say is hard about learning mathematics?

(f) How did they approach basic mathematics problems?

(g) How did these students define success in mathematics?

At the center of my conceptual framework is the narrative-identity framework developed by Sfard and Prusak (2005), who equate identity-building with storytelling. They argue that the stories people tell about themselves and their experiences are "words that are taken seriously and shape one's actions" (p. 21). Over time, these stories and the actions they inform come to define one's identity. The development of an identity helps us deal with change; it allows us to use our past experiences to plan for new situations with the expectation that "much of what we see now will repeat itself in a similar situation tomorrow" (p. 16). These identities are human-made based on messages we receive; these messages inform the stories we come to tell about ourselves. Sfard and Prusak developed their framework to answer two questions: "*Why do different*

individuals act differently in the same situation? And why, differences notwithstanding, do different individuals' actions often reveal a distinct family resemblance?" (p. 14, emphasis original).

In addition to their knowledge of mathematics, my participants brought identities, emotions, beliefs about mathematics and motivation with them to their developmental mathematics courses—all of which were profoundly shaped by their prior experiences and the messages they had received about themselves as math learners. I used this framework to explore the ways that these experiences informed their identities in the past and continue to shape their actions and understandings of themselves in the present. In doing so, I also found that other aspects of mathematics-related affect interacted with their identities and influenced their actions and ongoing identity development in a reciprocal relationship.

As I analyzed the data for my four focal-participants, I identified themes that capture important aspects of their learning and life experiences. I also noticed the ways that these themes were related to the learning and life experiences of my non-focal participants. These themes included: the development of strategies for self-protection; the influence of external and internal messages on identity development; the effect of the environment on learning and life experiences; the effect of accommodations on learning; persistence; and the reciprocal relationships among identity, attitude, emotions, motivations and the environment of learning and life experiences.

In this section, I look across my participants to better understand how they experienced these themes. I also include a section about what my participants had to say about mathematics itself and, in particular, what they reported is hardest about learning math. I did not explore their views of mathematics as a theme in the portraits because it was more meaningful collectively

than individually. I end this chapter by returning to my motivation for this study and my research question, and I relate these things to the questions raised by the authors of the narrative-identity framework that is central to my conceptual framework.

Development of Strategies for Self-Protection

While talking about their identities and past learning and life experiences, my participants also described various strategies they had developed to protect themselves from potential harm. I found this to be a dominant theme across my participants. The need for protection was commonly tied to their emotions, in particular fear and anxiety. These strategies were sometimes reactionary and sometimes intentional; some such strategies helped with learning while some hindered it. Participants' strategies developed out of their life experiences, both reflecting and shaping the way that they understood themselves as math learners.

The most commonly talked about self-protective strategies were those that came about as a reaction to fear of judgment. For example, based on his experiences, John demonstrated a strong sense that he would be negatively judged if he asked for help. He was often hesitant to ask for necessary and legally mandated accommodations for his disability, which hindered his learning. He later used the emotion of gratitude as a strategy to motivate himself to work hard so that he could graduate from high school.

Karen described intentionally developing apathy, as best they could, to deal with the pain of anxiety that arose from caring about math while not feeling successful at it during middle school. Although Karen stopped using apathy as a self-protective strategy during high school, their fatigue from an undiagnosed and untreated mental illness was viewed as a continuation of their earlier apathy and misinterpreted as laziness. Their parents' misinterpretation of their motivation led to being negatively judged when what they needed was medical help.

For Brad, fear of social isolation led him to develop the strategy of acting silly in class to get attention from his classmates. According to him, this led others to the misperceive him as having low intelligence and generated associated negative messages. Although Brad was able to maintain his self-view of being smart, the messages were painful. He eventually used a more positive strategy of studying and showing what he could do to push back on negative misperceptions. In addition, Brad was unique among my participants in needing additional strategies for protection from physical violence in school. He protected himself by keeping his jacket on throughout the day, every day, so that other students would not see how thin he was; he fought when necessary; and he had a group of students whom he relied on for protection.

Although my participants developed their self-protective strategies during learning experiences prior to college, these strategies may continue to shape how they understand themselves as math learners which in turn may impact the ways that they experience their developmental mathematics courses. This is especially concerning because, as Karen's and Brad's portraits illustrate so clearly, what others observe about a student may not be a true reflection of what the student is experiencing underneath. Strategies such as not asking for help to avoid negative judgment can be misinterpreted by an observer, such as an instructor, as disinterest, laziness, or lack of ability. Such misinterpretations can lead to negative messages from observers—as was the case for John, Karen, and Brad, for example. These messages can negatively influence the student's identity and further hinder learning.

The Influence of External and Internal Messages on Identity Development

Each of my participants received external messages came from a variety of in-school and out-of-school sources. For some participants, the in-school messages were strongest, while others received their strongest messages from home. Messages between home and school were aligned

for some participants, but for others, messages were in conflict. In addition, internal messages are those messages that participants told themselves as they worked on math and felt successful or unsuccessful, smart or not smart. They sometimes assessed themselves based on how well they felt they understood the math they were working on, but more often their self-assessments were based on getting correct answers after following rules. Some participants also assessed themselves in comparison to their peers.

For some, the external messages they received were consistent. But for others, these messages changed frequently. My participants received verbal messages from teachers, peers, tutors and counselors. They also received messages from test scores, prizes, and course placement. Messages from home were typically verbal ones from parents and occasionally from siblings. However, two participants, Sarah and Victoria, received messages from the ways that their families tried to engage them in math outside of school. Regardless of the source or consistency, these external messages combined and significantly influenced the ways that my participants came to see themselves as math learners.

Kelly, Ashley, Pearl and Elijah each remembered receiving mostly consistent external messages between home and school as well as from year-to-year. Further, their own self-assessments aligned with the external messages that they received. So, they did not need to make any decisions about which messages to accept. In each of their cases, the messages they received led to developing an identity as one who is not good at math. At the same time, their external messages also included encouragement to work hard and to ask for help. They found that doing so paid off. In each case, help was readily available. None of these participants reported feeling belittled or dismissed, and each developed a high level of comfort in asking for help. As a result, each developed a sense of self-efficacy that was distinct from their overall identity as a math

learner. In other words, each reported that they are capable of learning math with hard work, although they identify as not good at math.

A key difference among these participants and the influence of messages was that Ashley developed a strong dependency on receiving help. Unlike Kelly, Pearl, and Elijah, the messages she received often included the implication that she was not able to learn on her own. This message was so strong early in her life that she rejected later messages from high school teachers who tried to tell her that she did know how to solve problems on her own. Her identity was well-developed by the time she got to high school, and the new messages had very limited impact.

The messages that Brad remembers receiving were perhaps the most inconsistent. He received an array of external messages from different sources at different times, some positive, but most negative. He had to make choices about which messages to accept and which messages to reject throughout K-12, which was a very stressful process for him. In Brad's case, his ability to step back and understand the role that his environment played on his learning helped him to decide which messages to accept or reject. He came to rely on his own self-assessment and decided to reject the many negative messages he received and see himself as smart, instead.

Like Kelly, Ashley, Pearl and Elijah, Brad's self-efficacy was not aligned with his overall identity. For Brad though, the disconnect lay in the fact that he saw himself as someone who is smart and capable of *learning* math but not as one who could *do* math. He recognized that he had not been provided with the help he needed to make curricular transitions as he moved from school to school and that he was not provided opportunities to learn with appropriately challenging work, which he felt led to laziness. Thus, while he did not doubt that he could learn math when provided with adequate opportunities to do so—an assumption borne out by his experience during his senior year of high school—he also understood that he was currently

unable to do a fair amount of math because he had been denied the opportunities to learn how to do math.

Like Brad, Karen also received different messages, but their messages changed around key transition points, rather than the back-and-forth that Brad experienced. So, for Karen, decisions about which messages to reject and which ones to accept did not occur as frequently. Early in their education, when math was easy for them, Karen developed an identity as one who is very smart at math and came to associate finding math easy with being smart. As math became more difficult and the messages they received began to change during middle school, their previously developed association of easiness with smartness led them to accept negative messages and conclude that they were no longer smart in math. They also developed a self-efficacy as being unable to do math. Their parents were unable to convince them that that was not the case. And as they moved into high school, their parents began to send negative messages as well, as they interpreted Karen's poor performance in school as laziness and apathy. Thus, Karen's initial decision to reject the positive messages their parents tried to send them came to be reinforced as their parents, responding to their own interpretation of Karen's developing identity, shifted to more negative messages later on.

Sarah and Victoria both received significant messages about math and their relationships to it as a result of being given extra math books from family members during elementary school. But the different messages that accompanied these books and the ways family members tried to engage them in out-of-school math led to different feelings toward math and different senses of themselves as math learners. Sarah came to view math as a chore because her mother presented it to her that way, actually putting math practice on her chore list. Victoria, on the other hand, received math books as gifts from her uncle who was a math teacher and presented math to her

as a fun activity. She internalized his message that math is something enjoyable that she could excel at, and this message stayed with her throughout K-12. Both Victoria and Sarah brought these messages with them to college.

The extent to which my participants' identity development was influenced by internal versus external messages varied. When external messages were consistent across sources and across time, it appeared that my participants accepted them without question and these messages had strong influence on my participants' identity development. In these cases, their self-assessments may have been significantly influenced by the external messages. But when messages differed between home and school or changed over time, my participants had to make decisions about which ones to accept and which to reject. In these cases, their own self-assessments often became more influential.

See Tables B1 and B2 in the appendix for details on the relationship between self-efficacy and identity, and for a summary of identity development.

The Effect of the Environment and Life Experiences on Learning

When my participants talked about the ways that their past learning and life experiences influenced their learning, their talk centered on two major types of experiences: (a) those that directly impacted their chances for learning and (b) those that led to increased levels of anxiety, which interfered with either learning or testing. While all participants talked about anxiety at least to some degree, some like Victoria and Karen specifically talked about the ways that anxiety impacted them mentally, making it difficult to concentrate. Others, including John and Sarah, described how anxiety associated with fear of judgment made it difficult for them to ask for help. Brad was unique in experiencing anxiety related to concerns about his physical safety in school.

Nearly all of my participants described their elementary school years as warm and loving, but at the same time, unchallenging. This left them underprepared for a sharp increase in the level of difficulty of mathematics in middle school. Most also felt that the middle school environment was not as warm and supportive as elementary school. Even Kelly, who was homeschooled, noticed a distinct increase in the level of difficulty in the math curriculum between elementary and middle school. On the other hand, Elijah and Brad changed schools so frequently that the change between elementary and middle school was less memorable.

The environments in which they experienced in-school learning ranged from loving and supportive to abusive and toxic. Some were strongly supported at home, some were not supported at all, and others had mixed levels of support. Most of my participants started college directly after high school or after one year off. Some had bad experiences their first time in college and returned for additional attempts up to 10 years after initial enrollment. Some of them experienced health issues including mental and physical disabilities. Some participants experienced learning in different school districts with varying levels of learning opportunities and abuse of authority. My participants were sometimes ignored by teachers, talked to in condescending tones, or, in some cases, were well-supported. Some were provided with an appropriate level of challenge, others were not. Their experiences with disability accommodations also varied and I describe this in more detail in the next section.

The effects of these experiences on learning were also varied, even if on the surface the experiences appeared to be similar. A comparison between Brad and Elijah clearly illustrates how past learning and life experiences may have surface-level similarity but can be lived and processed very differently, leading to different identity development. Both are African American males who speak English as a first language, and neither have an identified disability. Both

moved frequently throughout their K-12 years and in each case some high school credits did not transfer. Neither are the first in their family to attend college. Both enrolled in Pre-Algebra as their first college math class. While neither used the word "trauma," both described negative experiences with lasting consequences that occurred during their K-12 education. On paper, or in large-scale quantitative studies of developmental mathematics, Brad and Elijah would appear to be nearly identical. However, their portraits revealed significant differences in how their past learning and life experiences had shaped them.

Elijah was raised by college-educated parents who supported his learning at home throughout K-12. Brad's mother went to college while he was in high school; he did not mention his father. So, although Brad and Elijah both answered "no" to the question "Are you first in your family to go to college?" Brad was not raised by college educated parents and did not receive any academic support at home. Conversely, strong support from home helped mitigate the impact of curricular misalignment on Elijah's learning, but he still felt that frequent school changes were "terrible," a feeling that Brad shared. Because of the differences in their experiences as well as the messages each received, they developed different self-efficacies, different overall identities, and different approaches to learning.

Elijah was 25 years old at the time of the interview and in college for the 4th time. The lasting impact of personal issues that began during high school led him to drop out the first two times. His 3rd attempt had been cut short because he had joined the Army and was called to active duty. He is determined to finish this time as the Army now pays his tuition. Brad was 21 years old at the time of the interview. He tried to enroll in college right after high school, but his financial aid did not come through and he had to leave. He reenrolled the following year. So, while both were unable to remain in college during their initial enrollment, they left for different

reasons and decided to return to college for different reasons. Although they appear the same on the surface, differences in their experiences and messages led them to develop a different sense of themselves as math learners with different learning approaches to learning which will likely lead to different responses to how they experience developmental mathematics as well.

Pearl and Victoria also changed schools, but they experienced their school changes in a positive manner. After being bullied at her first high school for being a good student, Victoria had the opportunity to choose to attend a regional technical high school for her junior and senior years which provided her a much more academically supportive environment than previously. Pearl started school in a public elementary school where she had limited access to much needed help. Her parents moved so that their daughter could have the benefit of a better education. In each case, school changes were intended to increase opportunities for learning in a more appropriate environment and had a positive impact on learning. Pearl did not initiate school changes, her parents did that for her, while Victoria choose to take advantage of an opportunity for greater academic rigor. This is in sharp contrast to the experiences Brad and Elijah had with school changes that they did not choose and had a negative impact on their learning.

So, the environment and life experiences can impact learning in ways that are not readily apparent and even have surface-level similarities and studies can have misleading results if details are not adequately captured. Changing schools during K-12 can have a significant impact on learning, but the impact on learning can vary. Simple yes/no questions about being first in family to attend college can also be misleading; it does not tell us if a student raised by college-educated parents, or by parents who can support learning. College attrition and time-off from college can be due to a variety of both academic and non-academic reasons.

The Effect of Accommodations Delivery on Learning

My participants' accounts of the accommodations they received for their disabilities suggest there is a range of support being provided in K-12. Levels of support they experienced ranged from accommodations being made readily available to accommodations that were only available upon request. Sometimes these supports for learning came with a condescending tone but at other times support was lovingly provided. My participants' experiences with in-school accommodations were also intertwined with the home environment. Taken together, these experiences shaped how some participants came to see themselves as math learners.

Students, such as John and Sarah, who received accommodations for a physical disability or as an English language learner, had negative experiences with their accommodations and felt that they were often talked to in a demeaning manner. Sarah, who has both a hearing disability and was an English language learner also felt that she was perceived as being less capable than she considered herself to be. However, she did not perceive a demeaning tone from friends when she asked them for help. The disconnect between self-perception and authority perception was a source of stress. For the most part, the participants who told me they had learning disabilities had accommodations that were more readily available. Ashley, who received unsolicited help from home, became overdependent on help, and never developed confidence in herself as one who cannot learn independently.

When accommodations were delivered only upon request, begrudgingly or with a tone of condescension, my participants felt anxiety, their identity formation was negatively impacted, and they developed self-protective strategies that had the potential to lessen learning. But when accommodations were provided with the message that those with learning differences need a

different type of instruction, rather than with judgement, Pearl developed a sense of pride in herself as one who is a hard worker.

Persistence

Most of my participants described ways that they have persisted in learning and moving forward with their education, although they did not use this term in the same manner as it is used in the literature. The literature defines persistence as progression from a developmental mathematics class to a course at the level of College Algebra or Pre-Calculus in a limited number of years, typically either three or six years. Instead, my participants described overcoming obstacles. When asked about what it means to be successful in math, responses ranged from understanding math to achieving certain grades in individual courses. Not one of my participants put a time limit on their degree completion.

But my data revealed a high level of persistence which came in many forms as many of my participants persisted in spite of having experienced barriers and unrecognized disabilities. Karen made very rapid progress in degree completion after they received a diagnosis and treatment. By the standards of the completion literature, Karen was a failure because 10 years had passed from time of initial enrollment until degree completion. However, they showed great persistence in overcoming obstacles, including paying tuition out-of-pocket while only working part-time, and they became a highly successful student by their own and their degree program's measure. Several of my participants showed persistence in retaking their first developmental mathematics course after not passing the first time. But the system does not recognize their progress.

What is Hardest About Learning Mathematics?

I didn't include what participants reported they felt was most difficult about learning math in the portraits or summaries because it did not seem as meaningful at the individual level as it does more collectively. However, as a general rule, when my participants told me what they struggled with, they told me they struggled with fractions. Some also described how math became difficult when "letters" became part of it (i.e., as they moved into algebra). In addition, Kelly, my homeschooled participant, reported that math became difficult when fractions became part of a problem in Algebra.

Brad felt that he could have learned fractions if he had help, but help was not part of his learning environment. Even though Elijah did receive a lot of help, he also said that the most difficult topic for him was fractions. Sarah told me she never got fractions. She learned how to use a calculator to work with fractions after elementary school. Her high school Algebra II teacher did not like fractions either, and did not required students to know fraction operations. Fractions are introduced in elementary school. Fractions and other topics that my participants mentioned are foundational to much of math throughout the remainder of K-12. If students do not develop an understanding of concepts that are presented during early schooling, it is likely that they will struggle with many topics in math until they do.

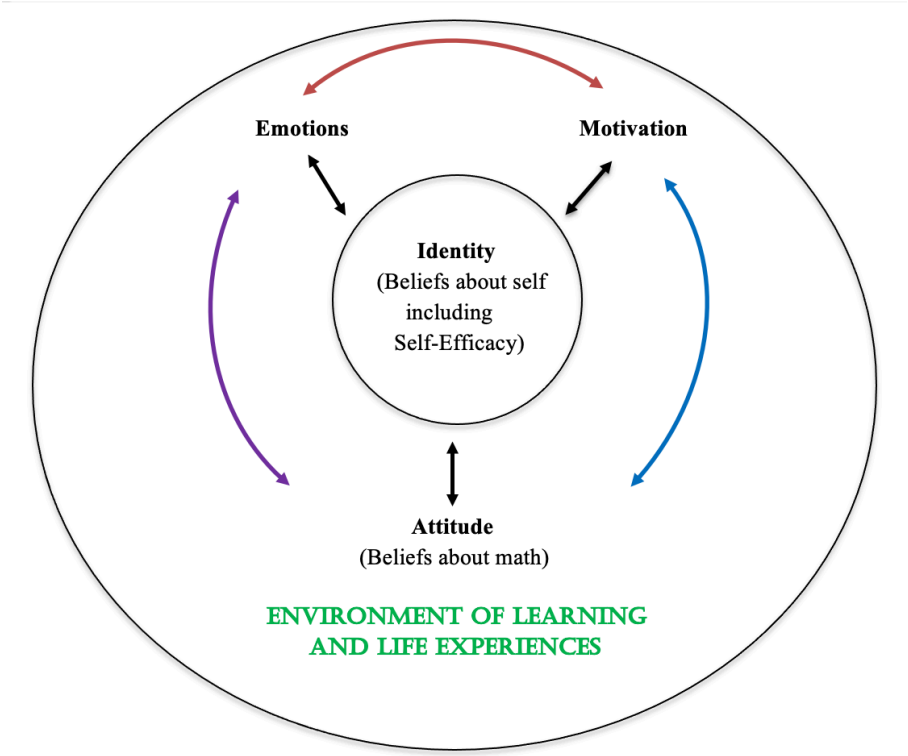
But looking beyond specific topics, when my participants talked about "doing" math, they talked about following rote procedures. Several participants specifically described that doing math is a matter of following steps and that math is difficult to remember. Many of my participants frequently talked about feeling the emotion of anxiety that interfered with testing and which was intensified by time limits. But several also described having anxiety in class as well.

Reciprocal Relationships Among Identity, Attitude, Emotions, Motivation and the Environment of Learning and Life Experiences

The environments in which my participants lived and learned were sources of messages that had the potential to impact their identities. These ranged from institutional messages, such as class placement, to verbal messages from teachers, counselors and parents. For instance, Brad received mixed identity messages from the different schools he attended. But messages from the environment also impacted participants' beliefs about math as well. For example, Sarah received messages, especially from her parents, that learning math is important for academic and career success. The environment also induced emotional responses such as anxiety and enjoyment. For John, at times, the school environment was a source of gratitude. Elijah's encouragement from home motivated him to work hard and to seek help with learning math, but personal issues at home also interfered with his ability to concentrate and learn.

However, rather than a unidirectional relationship between environment and affect, I found a reciprocal relationship among my participants' affective characteristics and their environments. Their identities, emotions, attitudes and motivations also influenced their environments, especially by influencing the ways that they were perceived by others. For instance, Brad identified as a brainiac when he attended a predominantly Black high school in which doing his work in class led to being mocked by students and even teachers. On the other hand, when Brad was in the predominantly White high school, he acted like the class clown as a self-protective strategy which led to an environment in which he was perceived as having low intelligence. Also, his motivation was often limited by limited opportunities offered by his learning environment, but when the environment changed during his last semester his motivation increased in response to a greater opportunity for learning.

Figure 5.1:
Reciprocity Among Aspects of my Conceptual Framework



There were reciprocal relationships among affective characteristics as well. In other words, identity, emotions, attitudes, and motivations shaped and reinforced each other, in addition to interacting with the environment. Many of these reciprocal relationships among affective characteristics revolved around self-protective strategies. For example, when Sarah asked for help with math at school, she felt negative emotions as tutors talked to her in a condescending manner. For a long while, these emotions reduced her motivation to ask for help. But, because of her attitude that math is essential for academic and career success, she was later motivated to push past those negative feelings and ask for help. In her case, the reciprocal relationship between her attitude toward math and her motivation eventually overrode the reciprocal relationship between her emotions and her motivation.

Each participant developed an identity as a math learner while they assessed their own learning and accepted or rejected messages they received. But their identities were also impacted by each of the other affective characteristics that they felt while they received these messages; the strength of their affective characteristics were mediated by their identities. For example, at one point, John was unmotivated to do homework because of low self-efficacy along with an overall identity as one who is not good at math. But this lack of motivation was temporary, and he became motivated to work hard because he believed in the importance of knowing math in life. At other times, he was unmotivated to request needed assistance because of a fear of judgment, but feelings of gratitude were eventually great enough to motivate him even as he continued to feel concerned about being negatively judged and continued to see himself as one who is not good at mathematics.

Sarah had a particularly complex relationship with math that developed along with her identity as a math learner. She began to develop the belief that math is not enjoyable and an identity as one who does not like math when extra math was presented as a chore when her primary learning environment was at home. However, in this environment she also developed the belief that math is important for academic and career success. She felt confused about math and unsupported in the school environment which influenced her identity development and conclusion that she is not good at math. So, although she had negative emotions and self-beliefs about math, believing it to be important motivated her to work hard, which showed her that she can be successful, and increased her self-efficacy.

So, although each aspect of mathematics-related affect had an in fluence on learning, none worked alone; each aspect influenced and had an influence on the others. Taken together, identity, emotions, attitude and motivation are more than the sum of their parts. Although my participants' presence and actions influenced their environment, their environment had a greater impact on them as individuals; each aspect of their affect was influenced by our system of education, i.e., the participants' experiences within their learning and life environments.

Answering My Research Questions

As I continue to address my main research question, I also consider the questions that were central to the narrative-identity portion of my framework. Prior to data analysis, I expected to find that my participants' identities as math learners, which they developed in their school and home environments, would reveal answers to the narrative-identity questions as well as my main research question. As I have previously described, I found that rather than identity alone, interactions among affective characteristics in conjunction with environment influenced my participants' actions. I slightly revised the narrative-identity questions to better reflect my particular study. I note the change from "why" to "how" with brackets:

> *Why do different individuals act differently in the same situation?*
>
> *And [how], differences notwithstanding, do different individuals' actions often reveal a distinct family resemblance?*

To answer these questions, I considered the key actions of my participants as math learners and identified the reasons for their actions based on my conceptual framework as shown in Table 5.1.

Table 5-1:
Relationship between Key Actions, Affective Characteristics, and Environmental Impact

Participant	Key Action(s)	Main Reasons for Actions Related to Framework
John	Avoided seeking help and asking for accommodations	Experiences, Emotion, and Identity: Fear of judgment; agrees with judgers that he is not good at it

	Stopped doing homework in middle school	Experiences, Emotion, and Identity: Despair; unable to do math; feeling unsupported
	Started doing homework again	Attitude and Identity: Belief that math is important; some low level of able to do math
	Reduced effort, almost dropped out of high school	Experience and Home Environment: Depression; impact of parents' divorce
	Started trying again	Emotions and Identity: Gratitude; maybe if I work hard
	Enrolled and reenrolled in developmental math	Experience, Motivation, and Identity: Needed for social work degree; hard work led to graduation
Karen	Engaged in math in school through 6th grade	Experiences, Emotion, and Identity: Joy of math; positive messages felt good
	Began to disengage during 7th & 8th grades	Experience, Identity, and Emotion: No longer smart; frustration and some apathy
	Disengaged during high school	Experience: Mental illness; lack of energy due to undiagnosed/untreated illness
	Enrolled and reenrolled in developmental math	Experience, Motivation and Identity: Needed for degree so they could change jobs; can do it
Sarah	Engaged with math at home as a chore during K-7	Experience and Emotions: Required by parents; wanted to please parents K-5; fear of being held back
	Worked hard, hesitant to ask for help from faculty, asked friends for help grades 8–10	Experience, Motivation, Emotions and Identity: Academic & career goals; fear of judgement; can learn with help
	Pushed herself to ask for help during later part of high school, grades 11–12	Experience, Attitude, Motivation, and Identity: Needs math for goals, can learn with help to learn
	Enrolled in developmental math	Motivation: Needed for degree
Brad	Watched public television	Emotion: Enjoyment
	Made people laugh at predominantly White schools	Experience and Emotion: Looking for social connections
	No effort at predominantly Black schools	Experience and Environment: No effort needed
	No effort at predominantly White schools	Experience and Environment: Did not get needed help with curricular changes; no study skills
	Began to put forth effort final semester of high school	Experience, Environment, Identity and Emotions: High expectations, capable, and smart; gratitude
	Enrolled and reenrolled in developmental math	Experience, Belief, Identity, and Motivation: Watched mother go to college; sees education as life changing; better life
Elijah	Worked hard, accepted and requested help	Identity and Life Experiences: Help was offered without condescending attitude, felt he needed it, felt it helped.
	Dropped out of college	Life Experiences: Lasting effect of personal issues

	Enrolled in developmental math	Emotion and Motivation: Transfer to HBCU to learn Black history
Victoria	Continued to participate in math class despite bullying	Emotion: Seeking teachers' approval
	Enrolled in tech HS for last two years	Motivation: Greater opportunity for learning, not necessarily due to math
	Enrolled and reenrolled in developmental math	Belief and Motivation: Needs math for academic and career goals
Kelly	Did not take Algebra II in HS	Life Experience: Not required or expected
	Worked hard, accepted and requested help	Identity and Life Experiences: Help was offered without condescending attitude, felt she needed it, felt it helped.
		Emotions and Identity: Proud of being a hard worker
	Enrolled and reenrolled in developmental math	Experience, Identity, Belief, and Motivation: Educated person
Mary	Blames teachers for low grades	Insufficient data
	Enrolled in developmental math	Emotion, Identity, and Motivation: Feels smart, needed for degree
Ashley	Worked hard, accepted help (No need to request)	Identity and Life Experiences: Help was offered without condescending attitude, felt she needed it, felt it helped.
		Emotion: Please parents
	Gave up on HW in 7th grade	Identity: Not good at it, cannot do it without help
	Relied on help	Experience and Identity: Cannot learn without help
	Enrolled in developmental math	Emotion and Motivation: Please parents.
Pearl	Worked hard, accepted and requested help	Identity and Life Experiences: Help was offered without condescending attitude, felt she needed it, felt it helped.
		Emotions and Identity: Proud of being a hard worker
	Dropped out of college the first time	Experience: Too difficult, not what she expected, did not take math
	Reenrolled, including taking developmental math	Experience and Motivation: Encouraged by employer for additional certification and higher pay.

Note: This table reflects the experiences and affective characteristics that had the greatest influence on key actions and is not meant to be inclusive of all aspects of my framework.

Why do different individuals act differently in the same situation?

The authors of the ICME-13 topical survey concluded that attitude is best defined as a construct of the observer (Hannula et al., 2016). Similarly, the apparent meaning of a situation may be a construct of the observer as well. In other words, two situations that may appear to an

observer to be the same, might, in fact, be quite different from the point of view of the individuals involved. My participants described memories of different past learning and life experiences which led them to develop different identities, attitudes, emotions, and motivation. As a result, different participants viewed similar situations through different lenses.

For example, when my participants found that they needed extra help with learning mathematics, their prior experiences with obtaining extra help varied and influenced their affective characteristics. As a result, they were not all inclined to seek help to the same degree. In the same vein, they experienced different levels of rigor and expectations as math learners, which led to differences in how hard each worked. Also, for some participants, working hard led to positive outcome, while for others, it led to frustration. These experiences impacted their motivation to continue to work hard in the future. Finally, those participants who were entitled to disability accommodations had different experiences and received different messages about themselves as math learners in that context, which led them to act differently with respect to accessing accommodations over time.

Because of differences in their past experiences as well as differences in the affective characteristics they developed as math learners, each of my participants have come to expect different outcomes of their actions. So, different participants acted differently in situations that may have appeared to be the same on the surface, because based on their points of view, they experienced these situations very differently.

And how, differences notwithstanding, do different individuals' actions often reveal a distinct family resemblance?

Nearly every one of my participants described strategies for self-protection that they developed, intentionally or unintentionally. Only my homeschooled participant did not describe a

strategy. The actions that they took to protect themselves were different and had different consequences for learning. See Table 4 in Appendix B for details. However, there is a strong family resemblance among their reasons for developing their strategies. Nearly all strategies were for the protection of their self-worth from the judgement of others. For example, Elijah expressed the dichotomy of learners who are "English people" and those who are "math people." In other words, he maintains that he is a worthy student and described himself as a good writer. He did so even at the risk of underestimating his mathematical ability. Ashley is quick to tell others she is not good at math to provoke a reassuring response that will affirm her self-worth. Mary attributed low grades solely to teachers. There were times when both John and Sarah avoided asking for help to avoid negative judgement. So, although they acted in different ways which on the surface appeared to be for different reasons, there was a strong family resemblance in the root cause which, overwhelmingly, was protection of their self-worth in the context of learning mathematics.

What do developmental mathematics students' accounts of their past learning and life experiences reveal about them as math learners that their grades do not?

My participants shared memories of a wide range of past learning and life experiences that profoundly impacted them as math learners prior to enrollment in developmental mathematics courses. They experienced a range of opportunities for learning and engaging in mathematics, highlighting everything from injustice and toxic environments to good teaching and loving environments. They also developed a range of identities, attitudes, emotions and motivations, including the motivation to protect themselves from perceived harm.

The literature mostly considers developmental mathematics in terms of the mathematical knowledge students bring. In other words, the literature assumes students either did not learn

enough math or did not learn it well enough. But, as my portraits reveal, the range of affective characteristics that students bring also impact their learning and may lead to actions that may be misinterpreted by parents, teachers and school support staff, further impacting their learning.

In the next chapter, I will discuss some implications of this study for instruction, institutional policy, future research, and public policy.

Chapter 6 : Implications

I decided to conduct this research study to gain a better understanding of who developmental mathematics students are as math learners. I had noticed a disconnect between the ways that these students and courses are portrayed in the literature versus what I observed in the classroom and during office hours. In particular, the data on student achievement in mathematics during K-12 highlight inequities in educational opportunities that begin long before students enroll in college and indicates that the majority of U.S. high school seniors are underprepared for college-level work. And yet, researchers often ignore the role of disparities in prior access and achievement. Despite these oversights, policy makers are beginning to limit developmental mathematics courses, (e.g., Visher et al., 2017), which has the potential to further widen gaps in mathematical knowledge.

I wanted to understand why some students, who from my view were intelligent and hard-working, had not learned the mathematics taught in developmental courses before enrolling in college. I also wanted to understand why some developmental mathematics students make large gains in learning in these courses while others do not. I designed and implemented a study to better understand how student learning may be impacted by their prior learning and life experiences and how those experiences may have shaped them as math learners

I collected data from 10 students who had enrolled in developmental mathematics. Eight of these participants were my own former students. Based on a questionnaire and interviews with participants, I found that I knew very little about my who my former students were as math learners and even less about their past experiences. I knew little of the emotions, identities,

beliefs and motivations that they brought with them. I learned that many of my participants had experienced toxic learning environments during their K-12 years in ways that I had not even imagined. They also experienced a range of emotions, including anxiety, hope, frustration and fear prior to their enrollment in their developmental mathematics courses. And they developed beliefs about the usefulness of mathematics and whether or not it can be enjoyed, as well as beliefs about themselves as math learners. It was humbling to learn about the high level of anxiety and fear of judgment that so many participants, including some of my own former students associated with learning mathematics. At the same time, I also learned about the ways that some of my participants demonstrated great persistence in ways that statistical data does not reveal.

In this chapter, I discuss the affordances and limitations of my study as well as the implications of my findings for instruction, research, and public as well as institutional policy.

Affordances and Limitations of My Study

My own prior learning and life experiences motivated me to conduct this study. But these experiences, including recognized and unrecognized privileges I have had, also have the potential to serve as a source of bias. In portraiture methodology, the need to look deeply at the data of each individual participant as well as build trusting relationships with them are ways researcher bias might shape the study. This necessary closeness may lead the researcher to make design decisions and/or interpret data in ways that reflect their personal biases toward participants. I addressed this potential limitation in my own study by carefully identifying and examining the potential biases I brought with me to the study at multiple points throughout the design, data collection, and analysis processes. However, though I examined my past experiences as they relate to this study, I must acknowledge that there may be areas of bias that I have not

recognized. In addition, I tried to avoid falling into the trap of thinking that I could put myself in my participants' shoes, recognizing that I have not experienced life and learning in the ways that any one of them have. I also tried not to impose my view of how they should act in any given situation because I have not been in their situation.

Including some of my own former students as participants was both an affordance and a limitation. It was an affordance because I was able to build on my existing relationships with them to gain the trust necessary for them to share their stories with me. I set aside my prior views of them as best I could during analysis, but at times I could not help but remember images I had constructed of them prior to this study. This might have led me to impose what I had observed about them which might not align with their own views of themselves. Coding close to the data, line-by-line, did help with this as I came to know them under their pseudonym in ways that I had not known them before. In the end, I feel that the benefits of including my prior students outweighed the limitations.

Another important consideration has to do with the ways that individuals construct and communicate stories about themselves. According to Sfard and Prusak (2005), a story-teller may tell different versions of their stories to different listeners: "Adjusting one's story to listeners is not a sign of insincerity but rather stems from the need for solidarity and effective communication" (p. 14). All participants, including those who were not my former students probably told me different versions of their stories than they would have told to another researcher. However, my findings do reflect a range of memories of past experiences, as well as a range of affective characteristics students bring with them to their mathematics classes; I expect that these kinds of variations would come through with another sample.

In addition, I do not have a representative or generalizable sample of developmental mathematics students. But I also do not know what such a sample would look like. Studies that claim to be based on nationally-representative samples of developmental mathematics students typically present these students according to their demographic data. However, I was seeking the type of data that cannot be found on a questionnaire. As illustrated by my comparison of Brad and Elijah, individuals who share important demographic markers may have very different learning and life experiences. Thus, although I cannot use my results to generalize to the whole population of developmental mathematics students, my findings do help the field begin to understand the range of who these students are as math learners, and the range of affective characteristics they bring with them to their mathematics classes.

Possibly the greatest limitation of this study is that my sample is made up of students who were willing to be interviewed about potentially sensitive issues, including four participants who did not pass their initial developmental mathematics class. This may suggest that these particular developmental mathematics students have a higher level of trust that our system of education can be improved. Specifically, Brad told me that he wants to tell his story in the hope that positive changes will be made.

The Contribution of Systemic Factors

As I described in Chapter 1, data from the National Center for Educational Statistics shows that only 3% of high school seniors scored in the advanced range on national tests and only 22% scored in the proficient range (McFarland, et al., 2018). According to these tests, it is highly likely that students below the 75th percentile—three-quarters of high school seniors tested—will not be ready for college-level mathematics. In other words, developmental mathematics students are ordinary students.

A primary takeaway from my study is that my participants' experiences reflect systemic problems in the U.S. educational system that profoundly shaped these ordinary students' paths to developmental mathematics. For instance, Brad's portrait illustrated the toxic environment he experienced at the schools he attended. Although he expressed a preference for the predominantly White schools he attended, he was not treated fairly or offered adequate opportunities to learn in either environment. The misalignment of curriculum and rigor that he encountered as he moved between schools are also structural problems. Each school/district set different expectations and offered different opportunities to learn, leaving students like Brad to navigate these differences on their own. While some students, like Elijah, may have access to educational support at home to help them manage these misalignments, Brad experiences illustrate that educators cannot assume that all students will have these resources to draw on.

Another systemic issue that hindered several of my participants had to do with the ways that schools construct students with disabilities and do or do not provide accommodations. For example, Sarah's teachers, support staff and counselors constructed an image of her as having limited intelligence because she was an English language learner with a hearing impairment; they regularly talked down to her and discouraged her from taking challenging courses long after she became fluent in English. Karen's experiences provide an example of how teachers and parents may construct students as lazy and unmotivated. Perhaps Karen's mental illness would have been diagnosed and treated sooner if authority figures in schools were not so content to judge and label them. In John's case, he should never have had to request his accommodations, they should have been provided automatically. For each of these participants, the ways that the school system viewed and treated them shaped how they came to view themselves as math

learners, which had material impacts on their ongoing learning and engagement with mathematics.

My participants' experiences reflect some of the ways that the U.S. educational system is set up to privilege some students over others and to provide inequitable access to mathematics education. Students who change schools frequently or who do not live in well-resourced school districts may not have the same access to consistent curricula and high-quality teaching that their more settled and/or affluent peers do. Students with disabilities may find themselves at the mercy of schools that fail to provide mandated accommodations and treat them as lazy or stupid for not performing in the same way as non-disabled peers. Black students and other students of color may be ill-served by both predominantly Black and predominantly White institutions.

My participants' experiences with systemic factors also illustrate the influence these factors can have on students' mathematics-related affective characteristics including identity development. For example, withholding approved disability accommodations until students advocate for them and treating disabled students in a demeaning manner negatively impacted the identities my participants with disabilities developed as math learners as they came to believe they are lesser-than their peers. Many of my participants also experienced negative emotions and came to not just fear but to expect judgment, which also influences their motivations and led them to devalue mathematics.

These systemic limitations on students' access, achievement, and mathematical affect and identity development in K-12 are likely to carry over to the college level. In the following sections, I discuss some of the ways that public policy and institutional policy could adapt to more fully account for the learning and life experiences that developmental mathematics students may bring with them to higher education.

Implications for Research and Public Policy

Research on Mathematics-Related Affect

Current research on mathematics-related affect tends to treat each aspect (i.e., identity, emotions, beliefs about mathematics, and motivation) in isolation and does not reflect the ways that they work together and change over time in relationship with one another and in relationship with students' environments. However, as I discussed in Chapter 5, these aspects of affect do work together and evolve as students experience opportunities and barriers to learning. As my participants' portraits demonstrate, different aspects of affect may be more influential on different individual's actions and learning than others because of their particular personalities and past experiences. These affective characteristics are as important to consider in research on developmental mathematical as the mathematical knowledge that students bring because of the ways that affect impacts future learning.

The field is in the beginning stages of research on developmental mathematics students. The affective characteristics that students bring with them to any mathematics class, including college developmental mathematics classes, are rooted in past experiences but will continue to evolve as they experience these classes. Future research should take into account that students' experiences in developmental mathematics is influenced by their past experiences.

Use of Demographic Data

In the previous chapter, I discussed how Brad and Elijah would be positioned identically in typical quantitative literature, even though their past educational experiences were quite different and have led to different identity developments. Although they appear similar on the surface—and show up almost identically in traditional demographic categories (e.g., gender, race, parental education)—key differences in their individual past experiences have led them to

develop different identities and expectations of the system. Thus, Brad and Elijah would likely respond differently to different interventions and approaches to instruction. Although he felt alone and unsupported in learning, Brad considers himself to be smart and put forth greater effort when more was expected of him during his last semester of high school. Someone like Brad might respond well to higher expectations and challenges in college. Conversely, although Elijah is very comfortable asking for help, he does not consider himself to be smart in math. Someone like him might respond well to more structured supports. Demographic data would not capture the ways that different students will respond to different types of instruction and other interventions.

Definition of Persistence

My findings also point to an important disconnect between the forms of persistence these students demonstrate and the forms of persistence that research on developmental mathematics research privileges. Much of the research defines persistence as completion of college-level courses within a limited time period. This definition frames students who do not meet these criteria as failures without considering these individuals' goals. My findings indicate that researchers' conceptions of persistence and success should be reconsidered in light of students' actual goals and life experiences. For example, only one of my 10 participants indicated a college-level mathematics class beyond Intermediate Algebra as their goal; one stated that Pre-Algebra was sufficient for their associate degree. Based on this, the majority of my participants would be categorized as failures in traditional research literature, even as they achieve their own educational goals.

Further, the time limits on course completion that researchers tend to impose fail to account for the actual experiences that students must navigate. Given the obstacles, that John has

experienced and overcome, passing Pre-Algebra in only two semesters was a significant accomplishment, not a failure. Karen struggled with a mental illness that interfered significantly with their learning. Yet, within two years of receiving a proper diagnosis and treatment, they completed a course in Intermediate Algebra, which fulfilled their mathematics requirement for their desired degree. Brad brought memories of an abusive and toxic school environment and may need time to heal while pursuing his academic goals. Artificially imposed time-limits do not allow for students who need to take time off for various reasons including mental or physical health, personal issues, and employment. So, many students who are positioned as failures in the quantitative literature are actually examples of persistence and success.

Research-Based Policy

Policy that is based on flawed research is bound to be ineffective. Much of the quantitative research that disparages developmental mathematics is based on the premise that developmental mathematics placement causes students to drop out. This is because some researchers have found a correlation between low retention rates and developmental mathematics placement and assume causation with no evidence. In response to this research, some states are limiting developmental mathematics course offerings, and access to federal financial aid has been restricted.

However, my analysis suggests that many of the same experiences that led my participants to developmental mathematics placements also led them to taking time off from college. For instance, Karen's undiagnosed mental illness both impeded their prior mathematical learning and led to their extended break from higher education. By the time they were able to return, they still needed the instructional opportunities afforded by developmental mathematics coursework. Neither time-limited restriction on financial aid access nor elimination of

developmental mathematics courses improved persistence or educational access. In determining definitions of success for the purpose of determining policy and for program evaluation, it is important for policy makers, researchers and college administrators to account for students' individual goals.

Implications for Institutional Policy and Practice

Although my participant had all been enrolled in a developmental mathematics course at the same community college within the same three-year time period, each brought an array of previously developed affective characteristics with them to their classes. They also brought an array of memories of past learning experiences. As a result of these individual differences, it is likely that they will experience any given developmental mathematics class differently. It is also likely that they will respond differently to different instructional techniques and interventions.

Use of Support Services

One question that I often had as an instructor was why students did not always take advantage of the support services available at the college. My participants' experiences provided a new lens for understanding this phenomenon and considering how instructors might address it. Based on their experiences, we might anticipate that students like Kelly, Pearl, or Elijah are well-situated to seek out assistance, such as tutoring centers or instructors' office hours, in ways that will support their learning in developmental math. However, these supports have the potential to hinder learning for someone who had developed an over-reliance on help, such as Ashley.

On the other hand, Brad grew-up feeling alone in learning. The system did not support him or recognize his potential until his last semester of high school. Someone like Brad may not see the system as there to support them, may not consider the possibility of seeking out supports,

and may not see supports as useful when he does encounter them. At the same time, someone who has been treated with condescension and negative judgment when seeking help, as John or Sara were, might avoid supports altogether.

At the course level, instructors should attend to the different ways that students might interact with support services and develop strategies to help students learn to take advantage of these resources. For example, an instructor might require all students to attend the tutoring center at least once per semester to help reduce the stigma sometimes associated with seeking help. At the institutional level, support services should be designed to be as accessible and non-threatening as possible. For instance, tutoring centers might train their staff to be sensitive to the affective needs of students like John and Sarah who may be more hesitant or skeptical about seeking help.

Disability Services

Simply knowing that a student received disability accommodations in the past does not tell a college instructor what their students' experiences were with those accommodations and how others' reactions to their disability may have impacted them as math learners. An important implication of my study is that the circumstances under which students receive accommodations are highly contextual and shape their mathematical affect. My analysis also suggests that students' accommodations are not uniformly provided in the K-12 setting. By law, students should not have to ask for their accommodations, but in reality, sometimes they do. It is important for instructors, administrators and support personnel to be aware that what students experienced in the past may influence their willingness to take advantage of accommodations at the college level. Additionally, developmental mathematics instructors should be aware that some of their students with disabilities may have had experiences that led them to feel both

stupid and anxious about their performance in math classes. Instructors should be aware of this possibility when interacting with these students and offer full compliance without hesitation or judgement.

In addition, community colleges have a responsibility to help instructors develop the practices they need to effectively enact accommodations. Instructors may need professional development provided by highly qualified individuals, about how to incorporate various types of accommodations into their regular practice so that when they do have a student with particular needs, they are ready and able to meet those needs.

Response to Instruction

Because students come to developmental mathematics with a range of past experiences and affective characteristics, it is likely that they will not all respond the same way to particular modes of instruction. This is part of what makes teaching these courses so challenging. Instructors should make concerted efforts to learn about their students as people and learners and use this information to implement various types of instruction within each class. Institutions should offer professional development opportunities to help instructors develop relevant strategies to do this work.

Moreover, it is easy for instructors and administrators to develop misperceptions of students when they view students through the lens of their own past experiences. In other words, it is easy to judge someone for not doing what we think we would do in their situation. But as my participants' portraits highlight, students' actions are likely shaped by factors that instructors do not know about. To address this reality, faculty should reflect on their own biases and privilege throughout the semester to understand that they do not know what students are actually thinking and feeling.

What is a Successful Developmental Mathematics Student?

Much of the research literature paints a picture of developmental mathematics students as failures based on the premise that all students should have goals that have been determined by the researcher. Instead, we need to listen to our students and ask them what their goals are and what they need to achieve those goals. As my participants' portraits illustrate, a successful student is one who works toward their goals within the reality of their current as well as past learning and life experiences. If a student needs to take a break from school for reasons such as employment, mental health concerns, physical health issues or family responsibilities, that does not make them a failure. Returning to college, even after a long break, should be recognized as persistence. A student who repeats a course or drops to a lower level course after realizing they had been over-placed, is persistence, not a failure. A student who achieves their personal and/or professional educational goals—even when those goals do not include "college-level" math—is a success, not a failure. Researchers, course instructors, and policymakers would do well to pay closer attention to how students define and pursue the goals they set for themselves.

Institutional Learned Helplessness

Developmental mathematics students are often described as being underprepared. However, as an instructor, I felt underprepared to teach these students. I have also found that institutions are underprepared to provide adequate supports with input from the students who need them. Despite the fact that developmental mathematics courses have been in place for well over a century, quality research aimed at improving these courses is limited. Institutions and instructors who work for them are instead led to believe that the *students* are the problem and that there's little we can do about this "reality" besides manage it. This kind of institutional

learned helplessness sets up a self-fulfilling cycle in which students do not receive the kinds of supports that would most benefit them and struggle in their coursework as a result.

I have found myself reflecting, somewhat unintentionally, on some interactions I have had with other former students that I did not understand at the time. I have thought about what I might have done differently had I known then what I know now. As an example, I had my first one-on-one conversation with a particular student somewhat late in the semester. Although he was very attentive and engaged when he did come to class, he sometimes missed and had not turned in any homework. His grade in the class was a "D" at that point. I was feeling a bit frustrated, so I said to him: "You are so smart. With full attendance and complete homework, you would have an 'A' in this class." He was speechless at first. I eventually realized that my construct of him as one who is smart in math did not match the way that he felt about himself. He did not feel that additional effort would change anything. I was underprepared to have a better follow-up conversation with him. I regret that I did not approach him earlier in the semester to understand how he was experiencing the class. I would urge instructors to get to know their students as math learners and to keep in mind that what they observe on the surface may not reflect the reality of what their students are thinking and feeling.

Developmental mathematics courses do have lower pass rates than higher level mathematics courses. It is easy for faculty and administrators to feel that nothing can be done to improve these courses and that it is up to the students to step-up and just work harder. Faculty and administrators cannot change the inequities and injustices in our K-12 educational system, but they have a responsibility to provide students with a new chance to learn in a meaningful way. Course elimination is not course improvement.

Appendices

Appendix A1: Consent Form (My Former Students)

Researcher: Carolyn Masserang
Doctoral Candidate in Mathematics Education
at the University of Michigan, Ann Arbor, MI
(248) 568 – 0806, cpmasser@umich.edu

Participant's preferred email:

The purpose of this form is to request your permission to participate in a research study I am conducting for my doctoral dissertation in mathematics education at the University of Michigan. I am interested in understanding the nature of students' experiences in learning mathematics during K-12 and how it impacts learning in developmental mathematics. The term developmental mathematics refers to mathematics courses taken in college below the level of pre-calculus. Research on these courses is limited, often based on students' transcript records, and paints a bleak picture. These studies do not tell us anything about who the students in these courses are or their history of learning mathematics. I would like to know more about students' learning experiences prior to college as well as during initial mathematics course(s) in college.

Your participation will consist of the following:

- Select a pseudonym to substitute for your actual name in the study.
- Fill out a written background questionnaire.
- Participate in one or possibly two 60-minute in-person interviews.
- Consent to having your grades and pre-tests from our class (with your name blocked out and replaced by your pseudonym) included in the study. You do not need to provide these. This is optional; you may refuse this and still participate in the study.
- Compensation is described on page 2 of this form.

If you agree to participate, I ask that you select a pseudonym in order to maintain confidentiality. I will only refer to you by that name in the study. I will only refer to this community college as "a suburban community college in the Midwest".

Please write your pseudonym: _____.

The study will consist of a written background questionnaire, and one or two interviews. I ask that you fill out the questionnaire in advance and either email it back to me or drop it off in the math office before the first interview. If this is not possible, please bring it to the first interview. I estimate that it will take 20 - 30 minutes to complete. Each interview will be no more than 60 minutes long. You may be asked to participate in a second interview within a few months of the first. If so, I will again ask for your permission. Interviews will be audio recorded and written out (transcribed) to help with my research, only. Other than myself, only members of my dissertation committee and possibly a transcription service will have access to the recordings and transcripts. You may review the transcription of your interview if you wish. The interviews will be semi-

structured, which means I will have some prepared questions, but we can deviate from them. You may go into more detail and may bring up things that you feel are relevant. You may ask for clarification of any questions and you may skip any questions that make you feel uncomfortable. This is also true for the background questionnaire. Because I value equity and inclusion, some of the questions will be about race/racism, gender, income, and disability status as far as you feel these things may have affected your learning. You may skip this if it makes you uncomfortable. I am hoping that you will share some stories about your experiences with me. During the interview, I will ask you to fill out a profile sketch describing how you see yourself as a learner of mathematics, and how you think others (such as teachers, classmates, family members, or other relevant people) see you. There will also be questions about particular mathematics problems. You may have already shared some learning experiences with me during our semester together. But, I know these things in confidence. In order for me to include them in my study, it is important that it comes from your questionnaire and interview responses.

At this time, please ask any questions or express any concerns that you have about the study.

Please indicate that you understand and consent to each of the following by signing your initials:

_____ I am at least 18 years of age.

_____ My questions about the study have been asked and answered and I may terminate the interview at any time.

_____ I consent to filling out a written background questionnaire questions which I will send back or bring to the first interview. I understand that I do not have to answer any questions that make me uncomfortable.

_____ I consent to an in-person interview which may last up to 60 minutes. I understand that the interview will be audio-recorded and transcribed under a pseudonym of my choosing. Only the researcher and I will know which pseudonym is associated with my actual name for the research study and I will be referred to by my pseudonym for research purposes.

_____ I understand that the interview will include questions about my experiences in- and out-of-school that relate to learning mathematics. In the spirit of equity and inclusion, some of the questions will be about how I feel that race/racism, gender, income, family background or disability status may have played a role in my learning.

_____ (optional) I consent to having my pre-tests (with my name blocked out and replaced by my pseudonym) and grades from the class I took with the researcher included as part of the study. The emphasis of the research is on learning, not grades, but these may be useful.

_____ I understand that I may or may not be asked to participate in a second interview and it will be my option to participate, if asked.

_____ I understand that for the background questionnaire and first interview combined, I will be paid $50 and that if there is a follow-up interview I will be paid an additional $50.

Appendix A2: Consent Form (not my students)

Researcher: Carolyn Masserang
Doctoral Candidate in Mathematics Education
at the University of Michigan, Ann Arbor, MI
(248) 568 – 0806, cpmasser@umich.edu

The purpose of this form is to request your permission to participate in a research study I am conducting for my doctoral dissertation in mathematics education at the University of Michigan. I am interested in understanding the nature of students' experiences in learning mathematics during K-12 and how it impacts learning in developmental mathematics. The term developmental mathematics refers to mathematics courses taken in college below the level of pre-calculus or college algebra. Research on these courses is limited, often based on students' transcript records, and paints a bleak picture. These studies do not tell us anything about who the students in these courses are or their history of learning mathematics. I would like to know more about students' learning experiences prior to college as well as during initial mathematics course(s) in college.

Your participation would consist of the following:

- Select a pseudonym to substitute for your actual name in the study.
- Fill out a written background questionnaire.
- Participate in one 60-minute in-person interview which will be audio recorded.
- Compensation is described on page 2 of this form.

If you agree to participate, I ask that you select a pseudonym in order to maintain confidentiality. I will only refer to you by that name in the study. I will only refer to this community college as "a suburban community college in the Midwest".

Please write your pseudonym: _____.

I ask that you fill out the questionnaire in advance and either email it back to me or drop it off in the math office before the interview. I estimate that it will take 20 - 30 minutes to complete. If it is not possible to turn it in ahead of time, please bring it to the interview but allow up to 15 minutes for me to look it over before the interview begins. The interview will be no longer than 60 minutes. Interviews will be audio recorded and written out (transcribed) to help with my research. Other than myself, only members of my dissertation committee and possibly a transcription service will have access to the recordings and transcripts. You may review the transcription of your interview if you wish. The interviews will be semi-structured, which means I will have some prepared questions, but we can deviate from them. You may go into more detail and may bring up things that you feel are relevant. You may ask for clarification of any questions and you may skip any questions that make you feel uncomfortable. This is also true for the

background questionnaire. Because I value equity and inclusion, some of the questions will be about race/racism, gender, socio-economic status, and disability status as far as you feel these things may have affected your learning. You may skip any of this that makes you uncomfortable. I am hoping that you will share some stories about your experiences with me. During the interview, I will ask you to fill out a profile sketch describing how you see yourself as a learner of mathematics, and how you think others (such as teachers, classmates, family members, or other relevant people) see you. There will also be questions about particular mathematics problems.

At this time, please ask any questions or express any concerns that you have about the study.

Please indicate that you understand and consent to each of the following by signing your initials:

_____ I am at least 18 years of age.

_____ My questions about the study have been asked and answered and I may terminate the interview at any time.

_____ I consent to filling out a written background questionnaire questions which I will send back or bring to the interview. I understand that I do not have to answer any questions that make me uncomfortable.

_____ I consent to an in-person interview which may last up to 60 minutes. I understand that the interview will be audio-recorded and transcribed under a pseudonym of my choosing. Only the researcher and I will know which pseudonym is associated with my actual name for the research study and I will be referred to by my pseudonym for research purposes.

_____ I understand that the interview will include questions about my experiences in- and out-of-school that relate to learning mathematics. In the spirit of equity and inclusion, some of the questions will be about how I feel that race/racism, gender, socio-economic status, family background or disability status may have played a role in my learning.

_____ I understand that for the background questionnaire and interview combined, I will be paid $50.

Appendix A3: Background Questionnaire

Researcher: Carolyn Masserang
cpmasser@umich.edu
(248) 568 – 0806

Pseudonym _____
(You may fill this in at the interview)

Please answer as many of the following questions as you feel comfortable answering. You may type your responses into this word document or print it and write out your responses using extra paper as needed. Please email your responses back to me or let me know if you will drop it off in the math office before the interview. If you find those options to be too difficult, please let me know and you may bring the completed questionnaire with you but allow an extra 15 minutes for me to look over the questionnaire before the interview begins.
Please note that these questions are the same for all participants, so some may not apply to you. If you have any questions or concerns about any of these, please contact me.

1. What type of schools did you attend during K-12? (For example, public schools, regional schools, charter schools, private schools, homeschool, etc.).

   Elementary School

   Middle School

   High School

High School:

2. Please list the math courses you took during high school.

3. Did you have a choice about which math courses you took?
   a. If yes, why did you choose these classes?

   b. If no, who or what determined your course selection? Would you have liked to have taken different courses? Explain.

c. At the time, how did you feel that your courses fit your learning? How has that feeling changed, if at all?

4. How challenged did you feel during each of your high school math courses?

   a. Do you like to be challenged in math? Why or why not?

5. How well do you feel your high school grades reflected your learning?

6. How were grades determined in your high school math classes?

   a. To what extent were math tests multiple choice?

   b. To what extent were your final grades determined by test, quiz and exam grades?

   c. To what extent were your final grades determined by homework and extra credit?

7. Standardized tests:
   a. Did you take the ACT _____, SAT _____, or both _____?

   b. Which other standardized tests did you take?

   c. To what extent do you feel these tests allowed you to show what you could do?

   d. To what extent do you feel your scores were a good reflection of your knowledge and understanding?

   e. Did you guess at many questions?

8. Did you have a job (or other time commitments) during high school?

   a. How many hours per week?

   b. What type of work did you do (or, what type of commitments did you have)?

9. When did you graduate from high school?

10. At what point did you decide to go to college?

   a. What influenced/motivated that decision?

After High School:

11. What are your long term educational and career goals?

12. What is the highest-level math course needed to achieve your goals?

    a. How do the required math courses help you with your goals (if at all)?

    b. What additional math (beyond requirements), if any, would be beneficial? Please explain.

13. First college math class:

    a. What math class did you initially place into in college?

    b. Was placement for your first class determined by a placement test?
    Yes _____, no _____.

        i. If not, how was placement determined?

        ii. If so, did you feel your placement was correct?

    c. Did you take the class you placed into, or did you take a different class?

    d. Do you feel a different first course would have been appropriate? Which one?

    e. How long after high school did you take your first college math class?

    f. How long after high school did you take any college class?

14. Please list college math classes you have taken or intend to take as indicated:

College math Course	Was it taken (do you intend to take it) at our college? Or at another 2-year or 4-year college.	When did you take (or intend to take) it? Specify semester/year	How prepared did you feel for this course?

15. College enrollment:
    a. Are you (or were you) a full-time or part-time college student?

    b. Are you currently enrolled at the same college as the one at which you were my student?

    c. Are you currently enrolled at another college?

16. Outside time commitments during college:
    Have you had a job during college? Full-time or part-time? What type of work?

    a. What other substantial time commitments (if any) did you have during college?

Math Questions

I am interested in understanding how you see yourself as a learner of math and what your prior learning experience was like as far as those things impact your prior and current learning.

17. Do you feel some people are just born to be "math people" and others are not?

    Do you feel that math is something that can be learned if taken seriously and with effort?

18. What math topic (if any) stands out to you as one that you thought you understood correctly during K-12 but you understand differently now?

19. How well did you feel you understood fractions prior to college?

20. Please explain how you would have added $\frac{1}{6} + \frac{2}{4}$ during K-12?

    a. Looking back now, how well do you feel you understood it?

    b. Please add $\frac{1}{6} + \frac{2}{4}$ without a calculator.

c. How has your understanding changed?

21. Please explain how you would have multiplied $\frac{3}{10} \bullet \frac{5}{6}$ during K-12.

a. Looking back, how well do you feel you understood it?

b. Please multiply $\frac{3}{10} \bullet \frac{5}{6}$ without using a calculator.

c. Please multiply: $\frac{x}{2} \bullet \frac{x}{5}$ =

d. How has your understanding changed?

22. Please simplify $12 - 3^2 + (-16) \div \sqrt{(25 - 9)}$ without using a calculator.

a. How well did you feel you understood order of operations during high school?

b. How has your understanding of order of operations changed?

23. At what point during K-12 did math become difficult (if it did)?

Background Information:

You have, no doubt, answered demographic questions like these many times on standardized tests. It is understandable if you are concerned about how this information will be used. I value equity and inclusion and would like to understand if and how your opportunity to learn has been impacted by the following identities. Please contact me if you have any concerns.

24. What is your age?

25. How do you identify yourself in terms of gender?

26. How do you identify yourself in terms of race/ethnicity?

27. Is English your first language? Yes _____, no _____. If not, what is your first language?

28. How do you pay for tuition?

    a. If you receive financial aid, do you feel it is adequate?

29. Are you first in your family to go to college?

    What effect has this had on your education?

30. Have you been identified as having a disability?

    a. Do you feel you have a disability? Please Explain.

    b. Have you at any time received accommodations?

    c. Do you feel your accommodations were helpful?

    d. How has your disability impacted your learning?

31. Have you at any time experienced racism, or discrimination, including microaggressions, due to race, gender, social class, disability or any other identities? Please explain.

32. Has your opportunity to learn been impacted based on any of these identities? Please explain.

Appendix A4: Interview Protocol

Research questions are listed here for reference only.

What do the past learning and life experiences of DM students reveal that their grades do not?
a. What obstacles and affordances to learning did a sample group of DM students experience while learning math prior to enrollment in DM? b. What narrative-defined identities did this sample of students develop as learners of math prior to enrollment in DM? c. How did the identities of these students shape their actions prior to enrollment in DM? d. How do these students now define success I mathematics?

Questions to be asked are numbered. Other text is explanatory.

1. Take a moment and think back to your earliest memories of doing math. Please tell me what you remember about it.

This serves as a warm-up for the first profile sketch.
Next, the participant is provided with a profile sketch. If the earliest memory is from pre-K (very unusual), start with a pre-K profile sketch. If the earliest memory is from the elementary school years, start with that.

Instructions for profile sketch:

2. There are stories we tell ourselves and others about who we are as a learner of math. We also hear stories and receive messages about ourselves from other people. These can be things someone tells us directly or can be messages we receive indirectly such as feedback on tests or things we overhear people say.

   This is a profile sketch. On the outside and to the left, please write about messages you received from others while you were in school during your elementary school years (or pre-K as appropriate) about who you are as a learner of math. On the outside and to the right, write about messages your received outside of school, such as at home or in your community. On the inside, write about the messages you told yourself and others about how you see yourself as a learner of math.

   Do you have any questions?

After allowing the participant time to fill in the sketch, ask about each thing they wrote. Begin with external messages for better flow into asking about how messages affected the participant.

3. I see here you wrote _____. Can you say more about that?

Provide additional prompts as needed.

Pausing and looking interested is an effective prompt for many participants.

Repeat this (question #3) for each item on the outside of the profile.

4. How were you affected by the messages you received?
5. How did these messages affect how empowered you felt to learn, if at all? Please explain.
6. Was there a time during _____(elementary school, etc.) when these messages changed? If yes, please fill out an additional profile sketch to reflect that.

Repeat question #3 for each item **inside** the profile.

Prompt as necessary.

The participant may have already talked about feelings toward math. If so, skip the next question. If the participant brings up feelings of anxiety, probe. If not, wait for the questions on anxiety near the end of the interview to ask, to give participants a chance to bring it up on their own, and to help avoid leading questions

7. How did you feel about math during _____(elementary school, etc.)?

The next question is to help understand how identity leads to actions, such as doing homework.

8. How much effort did you put into doing and learning math during _____ (elementary school)?

9. How did you feel about math homework?
    a. Did you feel it was worth doing?

10. How were grades determined?
    a. What messages did you receive from your grades about your identity?
    b. What messages did you receive from your grades about expectations of math students?

Messages are received from teachers and classmates:

11. What was your relationship like with your teachers during _____ (elementary school)? Why do think that was the case?

12. Were there particular teachers who made a difference in your learning?

13. What was your relationship like with your classmates?

Profile sketches for additional phases of learning:

14. Please fill out a profile sketch for _____ (middle school, high school).

Repeat the profile sketches and questions for the middle school years and for the high school years and repeat questions 3 – 13 as appropriate. If the student indicates a change in the type of messages within a particular level of schooling, an additional profile sketch may be appropriate. Also, the background questionnaire may indicate if an additional sketch is appropriate. For

example, an older student may have started college at a younger age, then stopped. If math class(s) were taken, an additional profile sketch can be done.

High School:
Begin with PROFILE SKETCH as above, but add

1. What kinds of instruction did you receive (pure lecture, interactive lecture, group work, worksheets)?
    a. What worked best for you? Why?

2. Were you taught to follow rules or to reason out solutions?

3. What is the hardest thing about learning math?

4. At what point did you decide to attend college? What influenced your decision?

5. Is there anything else about your k-12 math experience that you think is important?

Follow-up on answers to the background questionnaire:
Ask the participant to say more about responses on the background questionnaire.

For example:

1. I see you wrote _____. Please you say more about that.

2. (If the participant was homeschooled) Please tell me about your homeschool experience?

3. (In reference to listed high school classes) How do you feel these classes went?

4. If there was a delay between high school and college, please tell me about it.
    a. Had you wanted to start right away?

Etc.
Questions about RGCD are especially important. Be sure to follow-up on those.

Success:

1. What does success in math mean to you?

2. Tell me about a time that you felt successful in math.

3. How is success related to grades?
    a. To being quick?
    b. To learning?

4. What are some things you consider obstacles to learning?

Anxiety:

1. Let's talk about anxiety. During elementary school,
    a. Did you often feel anxious during math class? If not, how did you feel (happy, sad)?
    b. Did you often feel anxious during tests?

c. Did you often feel anxious while doing homework?

2. During middle school,
   a. Did this change?
   Same questions

3. During high school,
   Same questions

4. Do you feel anxiety affects your learning? How so?
   a. Your performance? How so?

5. What are you concerned about that makes you feel this way?
   a. What are you afraid will happen?

6. During college?
   Same questions

As time allows:
College:
Answers from background questionnaire and follow-up will affect some of these.
If the participant persevered through multiple courses, ask:
1. What motivates you to keep going in math?

If the participant did not persevere, ask:
2. Why did you stop taking math classes?

3. Describe how your college mathematics learning experience compares with what you experienced during high school?

4. What was the hardest thing about college math?

5. What kinds of things support your learning?

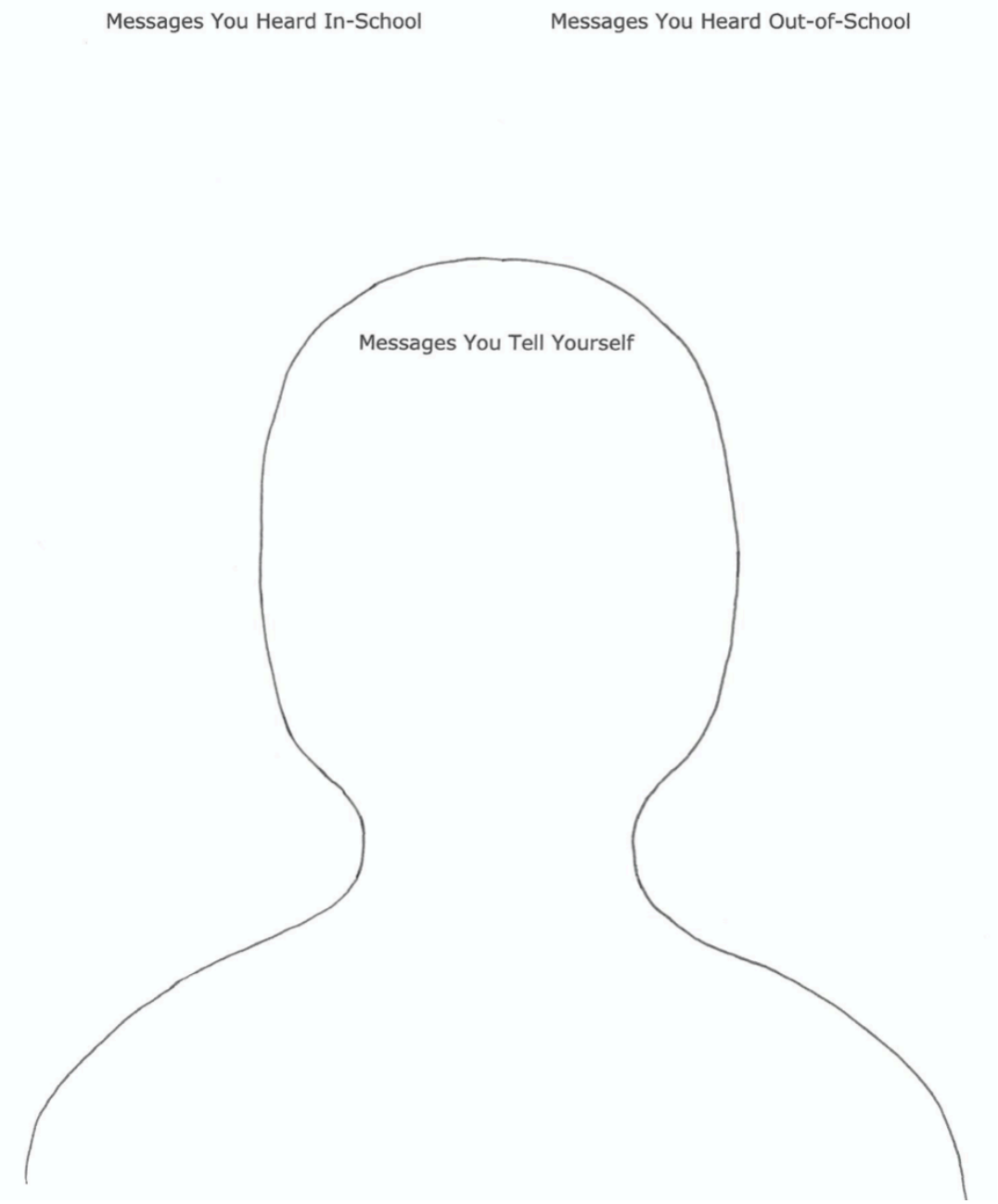
6. Is there anything about your college math experience that you would like to add?

Final Thoughts

7. Is there anything you would like to add?

Appendix A5: Profile Sketch

Appendix Figure 1:
Profile Sketch



Appendix B1: Identity and Self-Efficacy

Appendix Table 1:
Identity and Self-Efficacy

Partici-pant	Math Identity	Self-Efficacy	Overall Identity	Note:
Brad	Positive Smart	Distinct from identity as a math learner. Could have learned given the chance	Aligned with math identity	Maintained strong sense of ability in spite of negative messages, both direct and indirect. Understood that the environment impacts learning. Attributed achievement to lack of opportunity not intellect.
Karen	Changed, often Negative	Aligned with identity	Aligned with math identity	Identity as a math learner was connected with ease of learning. Smart means math is easy. Readily accepted "not smart' but pushed back on label of lazy. Obstacle of unrecognized and untreated mental illness. Successful reenrollment and achievement after treatment
Sarah	Negative But "not stupid."	Aligned with career options	Distinct: Math vs all else	Aligned with career options. Math is a chore. Felt strong in other subjects but never in math. Pushed back on negative messages; felt talked down to as "stupid". Accepted that she is not good at math but rejected negative messages about other subjects.
John	Negative Vicarious	Aligned with identity	Negative	Pre-diagnosis felt "pressured & stupid." Post-diagnosis felt like a burden. Felt behind his peers. Felt judged, felt like a nuisance 5th grade. Struggles with math.
Victoria	Positive Good with numbers	Aligned with identity	Positive	Receiving math books from uncle as gifts was significant. Bullied during Alg I & II for being a good student. Consistently felt strong with math. Hands-on learner; extreme extrovert; anxiety & feeling alone interferes with testing.
Kelly	Mildly negative	Distinct: Can do it with hard work.	Math vs all else	Homeschooled K-12 so messages were consistent. Struggled with timed tests. Better to be correct than fast. Not good at it, but with

	Not the best at math.			hard work can do it. Feels a sense of accomplishment after learning difficult topics.
Mary	Smart in math	Distinct at times Some Teachers make it hard.	Struggled with language arts	Believes rote memorization is math. Strong positive feedback during ES. Messages from rewards: stickers, pop, candy, accolades. Rejected all negative messages. When she does well it is because she is smart. But attributes struggle and low grades to bad teaching.
Ashley	Negative Never good at math	Can only do it with help	Negative	Ashley has always identified as someone who is not good at math. Unable to feel accomplishment when she does well. Diagnosed with a learning disability during elementary school. Strong support at home & school came with the message that she could not learn on her own
Pearl	Negative Learning disability, but proud of hard work	Needs strong support Can do some arithmetic	Negative	Identified as having a learning disability during elementary school. Always received strong support at home and received quality accommodations without feeling stigmatized since 5th grade. She feels that she cannot learn without strong support. Mediated by pride as hard worker
Elijah	Negative Terrible at math	Can succeed with hard work	Dichotomy He is an "English" person	Strong parental support. Messages of needing help and feeling struggle led to feeling he is terrible at math. Parents and sometimes teachers encouraged him to get help. Personal issues interfered with learning, but still came to see himself as terrible at math. Feels strong as a writer.

Appendix B2: Identity Development

Appendix Table 2:
Identity Development

Partici-pant	Source, Nature & Consistency of Significant Messages and Acceptance/Rejection of Messages
Brad	Brad often received mixed messages including indirect messages from his environment, teachers and peers such as being treated as "the class clown" in PWI and from an unsafe environment and being mocked for doing well at PBI. He understood that changing schools & misaligned curriculum without support from home along with low effort were the reasons for low grades at the PWI. He rejected negative messages; always considered himself to be smart & able when given the opportunity.
Karen	Karen's identity developed through four phases that transitioned at key points with messages they received, mental health, beliefs about math, and learning experiences. (1) Math is fun & easy, and I am smart; (2)I am not smart & don't care;(3) I have no energy but am not lazy, doing poorly in HS; (4)initial failure at CC before diagnosis & success. Messages & self-perception aligned during phase 1, misaligned during phases 2 & 3. Failing college math led to deep depression, hospitalization & diagnosis of mental illness, learned how to control symptoms, reenrolled & succeeded.
Sarah	Parents assigned extra math work in elementary school & presented math as a chore. They provided help at home during early years, but she was on her own after that. Received negative messages in school due to being an ELL and deaf in one ear; felt talked down to as "stupid." Condescending messages made it hard to ask for help. Accepted that she is not good at math but rejected negative messages about other subjects. Felt successful at home in elementary but never in school.
John	Pre-diagnosis: felt "pressured & stupid." Post-diagnosis: felt like a burden. Felt behind his peers, felt judged, felt like a nuisance in 5th grade. Few harsh teachers (T) had lasting impact even though most T were helpful. Family was supportive and helpful but encouraged independence and pushed him to advocate for his accommodations. Emotional support from HS choir T and twin sister. Struggles with math, feared being judged as stupid, embarrassed by low placement
Victoria	Received consistent positive messages. Receiving math books from uncle as gifts was significant. Bullied during Alg I & II for being a good student. Other messages were positive but not strong. Feels she must do math herself to understand it; listening alone does not work. Does better if she knows the teacher cares. Hands-on learner, "numbers came easily to me," "really strong with math." Extreme extrovert. Anxiety & feeling alone interfere with testing. Consistently felt strong with math.

Kelly	Mother was her only teacher during K-12 (homeschooled) so messages were consistent. Remembered messages in the form of advice: persevere until you understand, take a break & come back when you feel defeated, apply same reasoning to similar problems. Struggled with timed tests. Self-assessed—better to be correct than fast. Not good at math, with hard work can do it. Dichotomy of math vs all else. Confidence from success 2nd time in pre-algebra carried-over.
Mary	Strong positive feedback during elementary years. Messages from rewards: stickers, pop, candy, accolades. She holds on to seeing herself as good at math because it has always been her best subject. Negative message from struggles with other subjects including strong message from needing 2 English classes in 8th gr. Rejected all negative math messages and attributed struggles to teachers. When she does well it is because she is smart. But attributes struggle and low grades to bad teachers making it too hard.
Ashley	Always identified as not good at math; diagnosed as having a learning disability during elementary school. Strong support at home came with the message that she could not learn on her own. In-school messages from teachers were mixed, but the negative message from low placement & messages from home overrode positive messages from high grades. No sense of pride was expressed. Accepted strong message that she can only do math with help. Unable to feel accomplishment; gave her grandmother the credit when she did well.
Pearl	Pearl was identified as having a learning disability during elementary school. Always received strong support at home and received quality accommodations without feeling stigmatized since 5th grade. Cannot learn math without strong support but takes pride in being a hard worker.
Elijah	Was required to work on math with parents after school until parents could no longer help. Parents and sometimes teachers encouraged him to stay after school for help as well. Acknowledged that personal issues interfered with learning, but still came to see himself as terrible at math. Expressed dichotomy of learners as either math people or English people. Messages of needing help and feeling struggle led to feeling he is terrible at math and needs help, but good at English.

Appendix B3: Significant Learning and Life Experiences

Appendix Table 3:
Significant Learning and Life Experiences

Partici-pant	Significant Learning & Life Experiences (LLE) & Impact on Learning	Important Take-away
John	John experienced strong emotions along with his accommodations for low vision since 3rd grade. He often had to advocate for himself to receive the accommodations he was legally entitled to. He developed a strong fear of judgment from the ways he was sometimes treated when he asked for help. He felt embarrassed and feared judgment from peers over his placement. But he had strong support from his sister, parents and some peers & teachers.	Students who experienced shame when asking for help in the past may expect it to happen again in college.
Karen	Experienced low energy and depression as symptoms of unrecognized, undiagnosed mental illness. Unable to learn at her potential as symptoms progressed. Parents and teachers misinterpreted symptoms as laziness, but Karen felt that was not the case and intentionally tried to develop apathy to deal with the pain of failure.	Apathy can be an intentional way of dealing with the pain of caring about math when it is not going well.
Sarah	Felt teachers and staff talked down to her as an English language learner who is also deaf in one ear. Their condescending tone and low expectations lasted long after she became fluent. On the surface. Felt shamed when she asked for help.	Students may have been treated badly in the past for reasons that are not currently visible but feelings carry-over
Brad	Experienced inequity and social injustice prior to college enrollment. Frequently changed schools and experienced both PWI and PBI. Negatively and strongly impacted by changing learning environments, including abuse of authority.	Lasting mental stress from past abusive, racist learning environments, hidden by pleasant demeanor.
Victoria	Attended regular public schools K-10, changed by choice to a regional technical school for 11-12. She was bullied for being a good student at her first high school where she took Alg. 1 & 2. Negative peer influence from non-serious students at first HS. Has strong physical reaction with test anxiety.	High anxiety can play a significant role in testing even for one who identifies as smart

Kelly	Homeschooled K-12. Mother was teacher, siblings were peers. No grades—stayed with topics until proficient, no deadlines. Had very little Alg. 2. Now, grades provide valuable feedback and can be rewards. Kelly was homeschooled in a loving environment.	Grades provide useful feedback. But moving on even if topics are not learned can be problematic.
Mary	Mary struggled in all subjects except math throughout most of K-12. Learned math as rote memorization and appears unable to move forward as a result. Resists learning for understanding because rote is easier. When math does not go well, she blames the teacher for intentionally making math hard.	Attributes low grades and/or struggle to poor teaching as act of self-protection.
Ashley	Ashley received help at home and at school without asking throughout her K-12 education; diagnosed with a learning disability; received accommodations including special placement. She never had to ask for extra help which was built in. Help was often imposed on her at home. She developed a dependency and asked for help even if not mathematically needed.	Dependency developed over many years is not easily undone, may be impossible in 2 semesters.
Pearl	Received quality accommodations for learning disability from 5^{th} – 12^{th} grade with one-on-one support & good teaching. Highest high school math class included some pre-algebra but mostly arithmetic in 12^{th} grade. First time in college did not go well; dropped out; went to a career technical institute. Encouraged by employer to return to CC part-time for another certificate. Employer pays tuition which indicates confidence in her ability to succeed.	Having a high school diploma does not mean a student had Int. Alg. or even Elem. Alg. math was not the reason for the 10 year break & does not mean failure.
Elijah	Frequently changed schools but received strong support from home which helped with transitions. Personal issues at home during high school led to difficulty learning with lasting impact for many years. During 9^{th} grade he lowered his self-expectations (grades) to cope with personal issues. This is his 4^{th} time in college. Surface level similarities to Brad but very different on close examination.	Frequent school changes negatively impacted grades. Credit transfer and learning. Personal issues in the past continue to have a negative impact.

Appendix B4: Strategies for Self-Protection

Appendix Table 4:
Strategies for Self-Protection

Participant	Threat	Self-Protective Strategy or Need	Potential Consequences
John	Negative judgment and humiliation	Avoided asking for help. When he did push himself to ask, he developed anxiety. Use of gratitude push through tough times and deal with anxiety	Long term hesitation and anxiety Excessive gratitude for small things may show he does not value himself.
Karen	Failing grades & teacher remarks including "stupid." Accused of being lazy.	Tried to develop apathy to deal with pain of caring about math when it is not going well. Being too tired to fight accusations of laziness led to increased anxiety	Appearance of apathy may have increased perception of laziness.
Sarah	Condescending tone and low expectations were humiliating. Counselors tried to limit access to challenging coursework	At first, did not ask for help, then pushed herself to do so. Pushed back on counselors by taking honors and AP classes, but not in math.	Difficult to comprehend under the stress of humiliation Lower learning and grades.
Brad	Physical harm. Verbal abuse No friends due to school changes. Underestimated, not challenged or helped	Kept jacket on as first line of defense. Had to fight back from physical abuse. Internalized abuse led to anxiety Made people laugh Protection of self-worth.	Possible suspension. Abusive, racist learning environments, hidden by pleasant demeanor.
Victoria	Bullying, feeling alone and unsupported.	Looked for approval from teachers as she continued to participate in class, even while being bullied. Changed to a regional technical school for 11-12. Internalized feelings led to severe anxiety.	High anxiety can play a significant role in testing even for one who identifies as smart

Kelly	None mentioned.	Homeschooled in a loving environment. No grades, no deadlines. Moved at own pace Minimal threat, minimal need for self-protection. Still felt math got hard when fractions became part of the problem.	Had very little Algebra II. Did not get feedback from grades. New at getting grades in college.
Mary	Feeling highly successful with rote instruction led to being highly resistant to learning in a way that will lead to meaningful understanding.	Attributes low grades and/or struggle to poor teaching. Blamed teachers for intentionally making math too difficult when it did not go well. Math was her best subject and she needed to see herself as good at something.	Cannot move forward in learning math.
Ashley	Help was often imposed on her at home. Received help to the point of feeling spoonfed and not smart.	Constantly seeking affirmation. Tells others she is not good at math to get reassurance.	Developed a dependency and asked for help even if not mathematically needed. Unable to feel a sense of accomplishment.
Pearl	Minimal. Aware that she has difficulty learning that most students do not.	Takes pride in working hard to feel successful.	Provided motivation
Elijah	Personal issues interfered with learning with lasting consequences. Constant suggestions that he needed help led him to attribute learning difficulties to himself.	Expresses a dichotomy of math people vs English people as act of self-protection.	May be underestimating his ability Willing to work hard.

Appendix C1: Detailed Enrollment in Undergraduate Mathematics Courses

Appendix Table 5:
Undergraduate Mathematics Enrollment

From table S.2 CBMS Chapter 1: MATH & STATS ONLY – no computer science Enrollment (in 1000s) in Precollege vs all Undergraduate Mathematics Courses						
	2000	2005	2010	2015	2015 vs. 2010	2015 vs. 2000
4-year Precollege	219	201	209	253	Up 21.1%	Up 15.5%
4-year Intro level	723	706	863	1000	Up 15.9%	Up 38.3%
4-year Calculus Level	570	587	748	807	Up 7.9%	Up 41.6%
4-Year Advanced Level	102	112	150	154	Up 2.7%	Up 51%
4-year Subtotal Math only	1614	1607	1971	2213	Up 12.3%	Up 37.1%
4-year Prob. & Stats. Intro Level	136	148	231	253	Up 9.5%	Up 86.0%
4-Year P&S Upper Level	35	34	32	60	Up 87.5%	Up 71.4%
Total 4-year Undergraduate	1785	1789	2233	2526	Up 13.1% from 2010	Up 41.5%
**Comp. Sci.	123	57	77	68		
Total for Comparison	1908	1846	2310	2594		
**********						
2-year Precollege	763	965	1150	782	Down 32.0%	Up 2.5%
2-year Introductory level	274	321	368	445	Up 20.9%	Up 62.4%
2-year Calculus & Above	106	108	138	152	Up 10.1%	Up 43.4%
Other	130	187	231	259	Up 12.1%	Up 99.2%
Total Math Only	1273	1580	1887	1639	Down 13.2	Up 28.8%
2-year Prob. & Stats. Intro Level	74	117	137	280	Up 104.4%	Up 278.4%
Total 2-year	1347	1697	2024	1918	Down 5.2%	Up 42.4%

**********						
Total 4- and 2-yr Precollege	982	1166	1359	1035	Down 23.8%	Up 53%
Total 4- and 2-year Intro Level	997	1027	1231	1445	Up 17.4%	Up 44.8%
2-year Other	130	187	231	259		
Total Calc. Level & above – no P&S	778	807	1,036	1,113		
Intro level P & S						
Upper level P & S	35	34	32	60		
Total 4- and 2-yr All Undergraduate	3206	3564	4364	4589	Up 5.2%	Up 43.1%

Bibliography

ACT. (2017). *The ACT test user handbook for educators*. http://www.act.org/content/dam/act/unsecured/documents/ACT-UserHandbook.pdf

Adelman, C. (1999). *Answers in the tool box. academic intensity, attendance patterns, and bachelor's degree attainment.* Jessup, MD: ED Pubs. Retrieved from https://proxy.lib.umich.edu/login?url=https://www-proquest-com.proxy.lib.umich.edu/docview/62476017?accountid=14667

Adelman, C. (2005). Concluding messages: The toolbox revisited—Paths to degree completion from high school through college. *Journal for Vocational Special Needs Education, 28*(1), 31-56. Retrieved from https://proxy.lib.umich.edu/login?url=https://www-proquest-com.proxy.lib.umich.edu/docview/61833494?accountid=14667

AMATYC (2020). *The history of AMATYC 1974 – 1999.* Retrieved from https://cdn.ymaws.com/amatyc.org/resource/resmgr/history/amatychistory.pdf

Bahr, P.R. (2008). Does mathematics remediation work? A comparative analysis of academic attainment among community college students. *Research in Higher Education*, *49*(5), 420-450.

Bailey, T., Jenkins, D., & Leinbach, T. (2005). Is student success labeled institutional failure? Student goals and graduation rates in the accountability debate at community colleges. CCRC Working Paper No. 1. *Community College Research Center, Columbia University*.

Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, *29*(2), 255-270.

Blair, R. (2006). Beyond crossroads—implementing mathematics standards in the first two years of college. AMATYC.

Blair, R.M., Kirkman, E.E., Maxwell, J.W., & American Mathematical Society. (2013). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2010 CBMS survey*. American Mathematical Society.

Blair, R.M., Kirkman, E.E., & Maxwell, J.W. (2018). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: 2018 CBMS Survey*. American Mathematical Society.

Boylan, H.R., & White, W.G. (1987). The historical roots of developmental education: Educating all the nation's people. *Research in Developmental Education*, *4*(4), 1-4.

Boylan, H.R. (1988). The historical roots of developmental education. *Research in Developmental Education*, *4*(3).

Breneman, D.W., & Haarlow, W.N. (1998). Remediation in higher education. A symposium Featuring" Remedial education: Costs and consequences.". *Fordham Report*, *2*(9), n9.

Carnegie Foundation. (2015). Carnegie math pathways. Retrieved from http://www.carnegiefoundation.org/in-action/pathways-improvement-communities/

Cohen, A.M., & Brawer, F.B. (1996). The American community college. The Jossey-Bass Higher and Adult Education Series.

College Board. (2017). SAT suite of assessments annual report. Retrieved from https://reports.collegeboard.org/pdf/2020-total-group-sat-suite-assessments-annual-report.pdf

College Board. (2018). *SAT understanding scores*. Retrieved from https://collegereadiness.collegeboard.org/pdf/understanding-sat-scores.pdf

Committee on College Entrance Requirements. (1899). *Report of Committee on College Entrance Requirements July, 1899*. National Education Association of the United States.

Committee of Ten on Secondary School Studies. (1894). *Report of the Committee of Ten on Secondary School Studies: With the reports of the conferences arranged by the committee*. National Education Association of the United States.

Cox, R.D. (2015). "You've got to learn the rules": A classroom-level look at low pass rates in developmental math. *Community College Review*, 43(3), 264-286. Retrieved from

Cox, R.D., & Dougherty, M. (2019). (Mis)measuring developmental math success: Classroom participants' perspectives on learning. *Community College Journal of Research and Practice, 43*(4), 245-261. doi: doi:http://dx.doi.org.proxy.lib.umich.edu/10.1080/10668926.2018.1456378

Cullinane, J., & Treisman, P.U. (2010). Improving Developmental Mathematics Education in community colleges: A prospectus and early progress report on the Statway Initiative. An NCPR Working Paper. National Center for Postsecondary Research.

Educational Resources Information Center (2020). Retrieved from https://search.lib.umich.edu/databases/record/8311?query=ERIC

Fong, K.E., Melguizo, T., & Prather, G. (2015). Increasing success rates in developmental math: The complementary role of individual and institutional characteristics. *Research in Higher Education*, *56*(7), 719-749.

Fong, K., & Visher, M.G. (2013). Fast Forward: A Case Study of Two Community College Programs Designed to Accelerate Students through Developmental Math. *MDRC*.

Gholson, M.L., & Robinson, D.D. (2019). Restoring mathematics identities of Black learners: A curricular approach. *Theory Into Practice*, *58*(4), 347-358.

Hannula, M.S. (2020). Affect in mathematics education. *Encyclopedia of mathematics education*, (pp. 32-36). Springer.

Hannula, M.S., Di Martino, P., Pantziara, M., Zhang, Q., Morselli, F., Heyd-Metzuyanim, E., ... & A Goldin, G. (2016). *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions*. Springer Nature

Hodara, M., & Jaggars, S.S. (2014). An examination of the impact of accelerating community college students' progression through developmental education. *The Journal of Higher Education*, *85*(2), 246-276.

Hoyt, J.E., & Sorensen, C.T. (2001). High school preparation, placement testing, and college remediation. *Journal of Developmental Education*, *25*(2), 26.

Hu, S., Park, T., Mokher, C., Spencer, H., Hu, X., & Bertrand Jones, T. (2019). Increasing momentum for student success: Developmental education redesign and student progress in Florida.

Hu, S., Park, T. J., Woods, C. S., Tandberg, D. A., Richard, K., & Hankerson, D. (2016). Investigating developmental and college-level course enrollment and passing before and after Florida's developmental education reform. REL 2017-203. *Regional Educational Laboratory Southeast*.

Huang, M., Hoang, H., Yesilyurt, S., & Thorn, C. (2016). Community college pathways: 2014-2015 impact report. Community College Pathways Technical Report. *Carnegie Foundation for the Advancement of Teaching*.

Humes, E. (2006). *Over here: How the GI Bill transformed the American dream*. Houghton Mifflin Harcourt.

Kane, T., Boatman, A., Kozakowski, W., Bennett, C., Hitch, R., & Weisenfeld, D. (2020). Is college remediation a barrier or a boost? I Evidence from Tennessee. *Education Next*, *20*(2), 64-71.

Katznelson, I. (2005). *When affirmative action was white: An untold history of racial inequality in twentieth-century America*. New York, NY: W.W. Norton.

Kozeracki, C. A. (2002). ERIC review: Issues in developmental education. *Community College Review*, *29*(4), 83-100.

Lawrence-Lightfoot, S. (1983). *The good high school: Portraits of character and culture*. Basic Books.

Lawrence-Lightfoot, S., & Davis, J. H. (1997). *The art and science of portraiture*. San Francisco, CA: Jossey-Bass.

Martin, D. B. (2013). Race, racial projects, and mathematics education. *Journal for Research in Mathematics Education*, *44*(1), 316-333.

Maxwell, J. A. (2013). *Qualitative research design: An interactive approach* (vol. 41). Sage publications.

McFarland, J., Hussar, B., Wang, X., Zhang, J., Wang, K., Rathbun, A., Barmer, A., Forrest Cataldi, E., and Bullock Mann, F. (2018). *The condition of education 2018* (NCES 2018-144). U.S. Department of Education. Washington, DC: National Center for Education Statistics. Retrieved from https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2018144.

McMillan, V. K., Parke, S. J., & Lanning, C. A. (1997). Remedial/Developmental education approaches for the current community college environment. *New Directions for Community Colleges,* (100), 21-32. doi: http://dx.doi.org.proxy.lib.umich.edu/10.1002/cc.10002

Melguizo, T., Bos, J. M., Ngo, F., Mills, N., & Prather, G. (2016). Using a regression discontinuity design to estimate the impact of placement decisions in developmental math. *Research in Higher Education*, *57*(2), 123-151.

Melguizo, T., & Ngo, F. (2020). Mis/Alignment between high school and community college standards. *Educational Researcher*, *49*(2), 130-133.

Merisotis, J. P., & Phipps, R. A. (2000). Remedial education in colleges and universities: What's really going on? *The Review of Higher Education*, *24*(1), 67-85.

Merriam, S. B. (2009). Qualitative case study research. *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Merseth, K. K. (2011). Update: Report on innovations in developmental mathematics--Moving mathematical graveyards. *Journal of Developmental Education*, *34*(3), 32.

Mesa, V. (2017). Mathematics education at U.S. public two-year colleges. In J. Cai (Ed.), Compendium for research in mathematics education (pp. 949–967). Reston, VA: NCTM.

Meserve, B. E., & Schumaker, J. A. (1957). Chapter VI: College mathematics and teacher education. *Review of Educational Research*, *27*(4), 375-390.

Miami Dade College (2019). Retrieved from https://www.mdc.edu/wolfson/departments/math/courses/descriptions.htm

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook*. 3rd edition. Sage Publications, Inc.

National Center for Educational Statistics. (2020a). *Digest of education statistics.* Retrieved from https://nces.ed.gov/programs/digest/d19/tables/dt19_306.20.asp?current=yes

National Center for Educational Statistics. (2020b). *Digest of education statistics.* Retrieved from https://nces.ed.gov/programs/digest/d17/tables/dt17_301.20.asp

National Center for Educational Statistics. (2020c). *Digest of education statistics.* Retrieved from https://nces.ed.gov/programs/digest/d19/tables/dt19_303.70.asp

Ngo, F., & Kosiewicz, H. (2017). How extending time in developmental math impacts student persistence and success: Evidence from a regression discontinuity in community colleges. *The Review of Higher Education*, *40*(2), 267-306.

Ngo, F., & Melguizo, T. (2016). How can placement policy improve math remediation outcomes? Evidence from experimentation in community colleges. *Educational Evaluation and Policy Analysis*, *38*(1), 171-196.

O'Quinn, R.L. (1931). High School and College Mathematics. *Mathematics News Letter*, 6–11. Louisiana State University

Payne, E.M., & Lyman, B.G. (1996). Issues affecting the definition of developmental education. *Defining developmental education: Theory, research, and pedagogy*, 11-20.

Reeve, W.D. (1946). Coordinating High School and College Mathematics. *The Mathematics Teacher*, *39*(8), 354–364.

Rotman, J. (2013). Inside new life: A grand vision for developmental mathematics. *MathAMATYC Educator*, *4*(3), 27-35.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.

Shalaby, C. (2017). *Troublemakers: Lessons in Freedom from Young Children at School*. The New Press.

Sitomer, A., Strom, A., Mesa, V., Duranczyk, I. M., Nabb, K., Smith, J., & Yannotta, M. (2012). Moving from anecdote to evidence: A proposed research agenda in community college mathematics education. *MathAMATYC Educator*, *4*(1), 35-40.

Stigler, J.W., Givvin, K.B., & Thompson, B.J. (2010). What community college developmental mathematics students understand about mathematics. *MathAMATYC Educator*, *1*(3), 4-16.

Valentine, J.C., Konstantopoulos, S., & Goldrick-Rab, S. (2017). What happens to students placed into developmental education? A meta-analysis of regression discontinuity studies. *Review of Educational Research*, *87*(4), 806-833.

Van Manen, M. (2016). *Researching lived experience: Human science for an action sensitive pedagogy*. Routledge.

Visher, M., Cerna, O., Diamond, J., & Rutschow, E.Z. (2017). Raising the floor: New approaches to serving the lowest-skilled students at community colleges in Texas and beyond. *MDRC*.

Williams, W.L. (1954). What the Colleges are Doing About the Poorly Prepared Student. *American Mathematical Monthly,* 86-88

Williams, D.E., & Siwatu, M.S. (2017). Location of developmental/remedial coursework predicts successful completion of college algebra: A study of Louisiana's developmental students. *Educational Research Quarterly*, *40*(4), 23.

Wyatt, M. (1992). The Past, Present, and Future Need for College Reading Courses in the U.S. *Journal of Reading, 36*(1), 10-20. Retrieved September 14, 2020, from http://www.jstor.org/stable/40032179